

The Galaxy–Dark Matter Connection

cosmology & galaxy formation with the CLF



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Outline

- **Galaxy Bias & The Galaxy-Dark Matter Connection**
- **Halo Bias & The Halo Model**
- **Halo Occupation Statistics**
- **The Conditional Luminosity Function (CLF)**
- **The Universal Relation between Light and Mass**
- **Cosmological Constraints**
- **Galaxy Groups**
- **Galaxy Ecology**
- **Satellite Kinematics**
- **Brightest Halo Galaxies**
- **Pairwise Velocity Dispersions**
- **Environment Dependence**
- **Conclusions**

Galaxy Bias

Consider the distribution of matter and galaxies, smoothed on some scale R

$$\delta(\vec{x}) = \frac{\rho(\vec{x}) - \bar{\rho}}{\bar{\rho}}$$

$$\delta_{\text{gal}}(\vec{x}) = \frac{n_{\text{gal}}(\vec{x}) - \bar{n}_{\text{gal}}}{\bar{n}_{\text{gal}}}$$

Mass distribution

Galaxy distribution

- There is no good reason why **galaxies** should trace **mass**.

- Ratio is **galaxy bias**: $b(\vec{x}) = \delta_{\text{gal}}(\vec{x}) / \delta(\vec{x})$

- One can distinguish various **types** of bias:

linear, deterministic: $\delta_{\text{gal}} = b \delta$

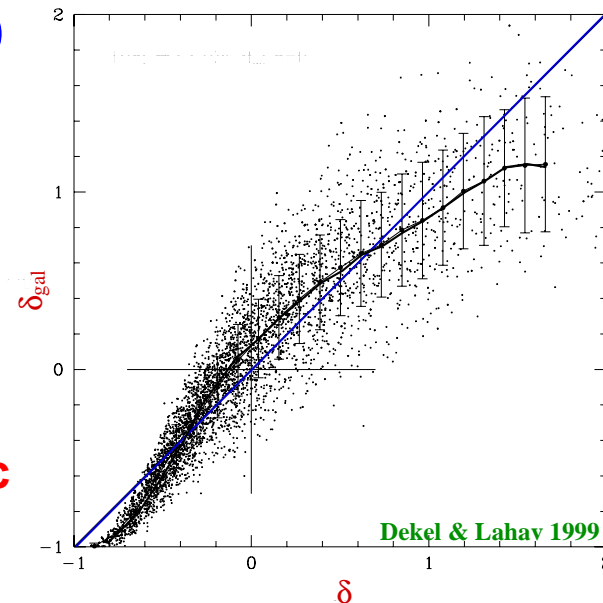
non-linear, deterministic: $\delta_{\text{gal}} = b(\delta) \delta$

stochastic: $\delta_{\text{gal}} \neq \langle \delta_{\text{gal}} | \delta \rangle$

- Real bias probably **non-linear** and **stochastic**

- Bias also depends on **smoothing scale** R

- Since $\delta_{\text{gal}} = \delta_{\text{gal}}(L, M_*, \dots)$ bias also depends on **galaxy properties**



Handling Bias

- Bias is an imprint of **galaxy formation**, which is poorly understood
- Consequently, little progress constraining cosmology with **LSS**

Q: **How can we constrain and quantify galaxy bias in a convenient way?**

Handling Bias

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Q: **How can we constrain and quantify galaxy bias in a convenient way?**

A: **With Halo Model plus Halo Occupation Statistics!**

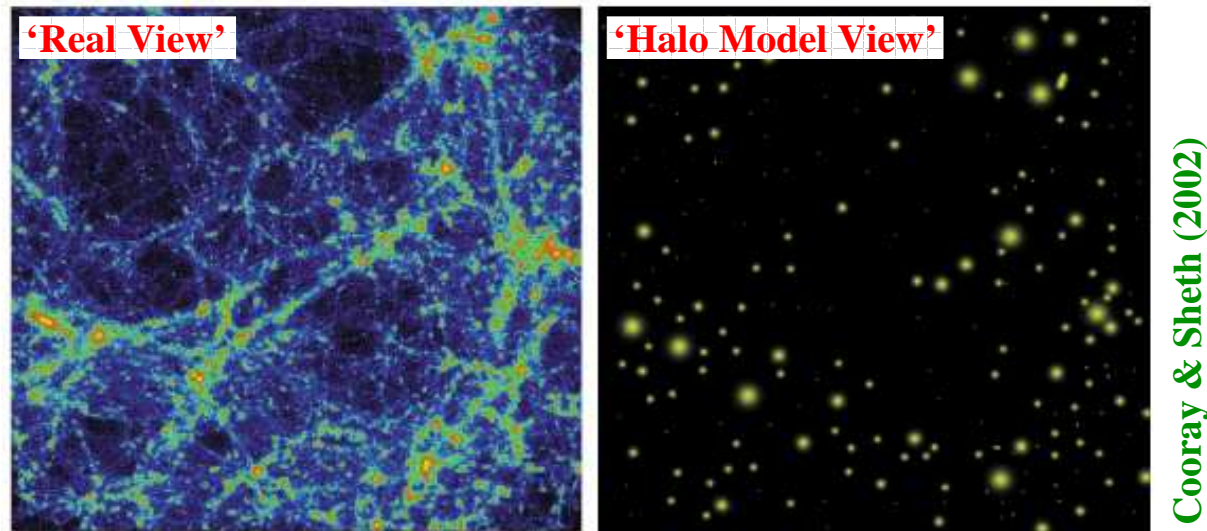
The **Halo Model** describes CDM distribution in terms of **halo building blocks**, under assumption that **every** CDM particle resides in virialized halo

- **On small scales:** $\delta(\vec{x})$ reflects density distribution of haloes (**NFW profiles**)
- **On large scales:** $\delta(\vec{x})$ reflects spatial distribution of haloes (**halo bias**)

PARADIGM: All galaxies live in Cold Dark Matter Haloes.

galaxy bias = halo bias + halo occupation statistics

Halo Model Ingredients



Halo Density Distributions: (Navarro, Frenk & White 1997)

$$\rho(r) = \frac{\rho_s}{(r/r_s)(1+r/r_s)^2}$$

Halo Mass Function: (Press & Schechter 1974)

$$n(m) = \sqrt{\frac{2}{\pi}} \frac{\bar{\rho}}{m^2} \left| \frac{d \ln \sigma}{d \ln m} \right| \sqrt{\nu} e^{-\nu/2}$$

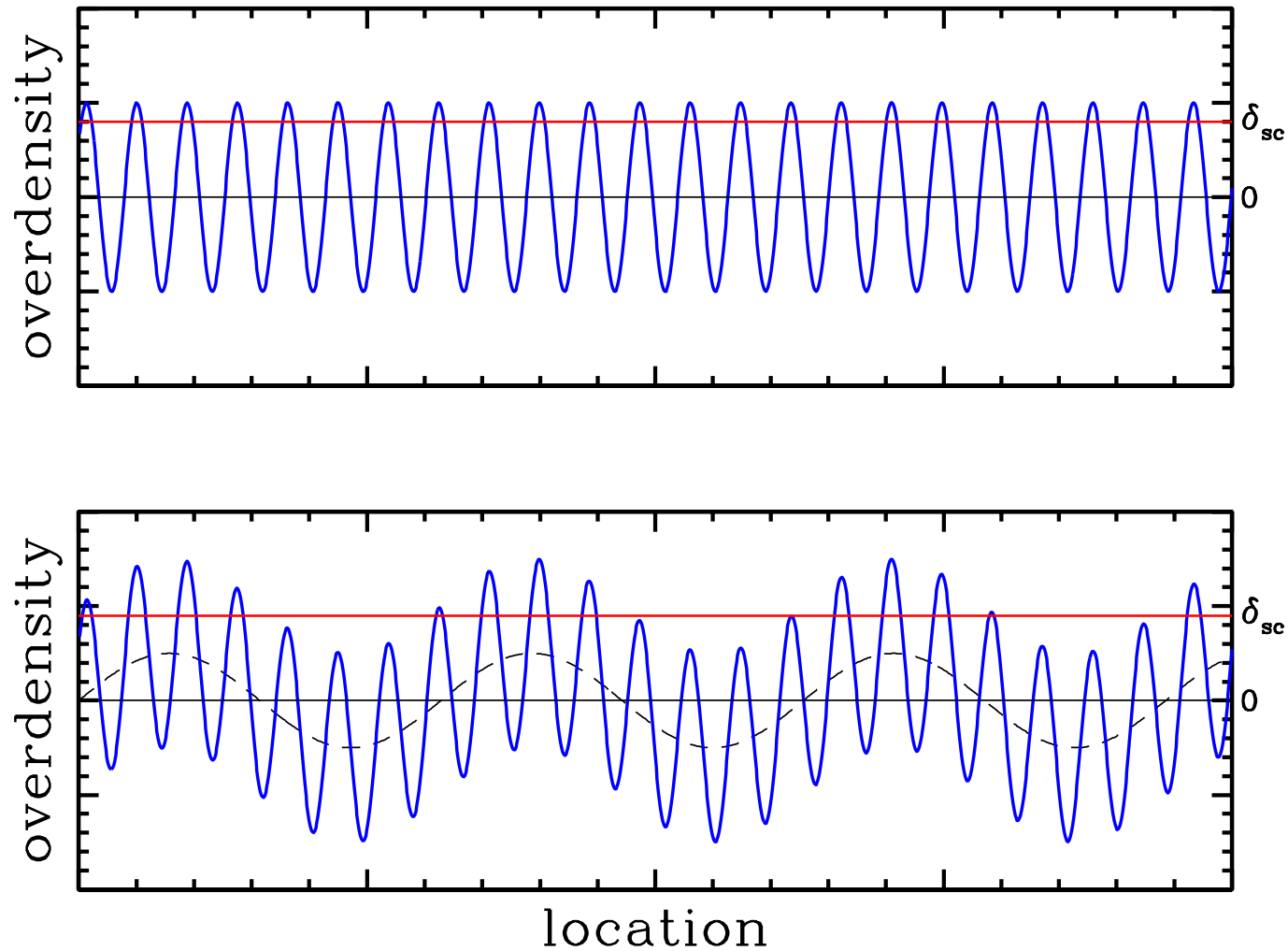
Halo Bias Function: (Kaiser 1994; Mo & White 1996)

$$b(m) \equiv \frac{\delta_h(m)}{\delta} = \frac{n(m|\delta) - n(m)}{n(m) \delta} = 1 + \frac{\nu - 1}{\delta_{sc}}$$

δ_{sc} is critical **spherical collapse** overdensity, $\sigma^2(m)$ is **mass variance**, and $\nu = \delta_{sc}^2 / \sigma^2(m)$

Statistical Halo Bias

Dark Matter Haloes are a **biased** tracer of the dark matter mass distribution!



Modulation causes **statistical** bias of peaks (haloes)

Dynamical Halo Bias

Halo bias has a **statistical** component plus a **dynamical** component

Growth of ‘modulation-perturbation’ causes increase in number density of haloes in overdense regions:

Consider a volume V with mass M , and let $M = V\bar{\rho}(1 + \delta)$

Here δ is the **overdensity** of V

At **very early** times this mass was in a volume V_0 for which $M = V_0\bar{\rho}$

This basically reflects that $\delta_{\text{init}} \ll 1$

$$\Rightarrow \boxed{V(1 + \delta) = V_0}$$

This reflects the **dynamical** biasing in the linear regime.

Derivation of Halo Bias I

Define **halo bias** as $b(m) = \delta_h(m)/\delta$

Let $N(m|M, V)$ be the number of haloes of mass m in volume V .

The volume V has an overdensity δ so that $M = V\bar{\rho}(1 + \delta)$ and initially was associated with a volume $V_0 = V(1 + \delta)$.

The overdensity in the **number** of haloes of mass m is

$$\delta_h(m) = \frac{N(m|M, V)}{n(m)V} - 1$$

Here $n(m)$ is the (average) halo mass function.

To take account of the **dynamical** bias we write

$$N(m|M, V) = n(m|M, V)V_0 = n(m|M, V)V(1 + \delta)$$

so that

$$\delta_h(m) = \frac{n(m|M, V)}{n(m)}(1 + \delta) - 1$$

Derivation of Halo Bias II

PS ansatz: haloes are associated with regions with $\delta > \delta_{\text{sc}}$

Therefore, we can compute $n(m|M, V) = n(m|\delta)$ by simply replacing δ_{sc} with $\delta_{\text{sc}} - \delta$ (**Peak-Background split**).

Using that the **halo bias** is defined as $b(m) = \delta_h(m)/\delta$, one obtains that

$$b(m) = 1 + \frac{\nu-1}{\delta_{\text{sc}}}$$

where $\nu = \nu(m) = \delta_{\text{sc}}^2/\sigma^2(m)$

Using that $\sigma(m^*) \equiv \delta_{\text{sc}}$ we see that

$$b(m) > 1 \quad \text{if } m > m^* \quad \text{(biased)}$$

$$b(m) = 1 \quad \text{if } m = m^* \quad \text{(unbiased)}$$

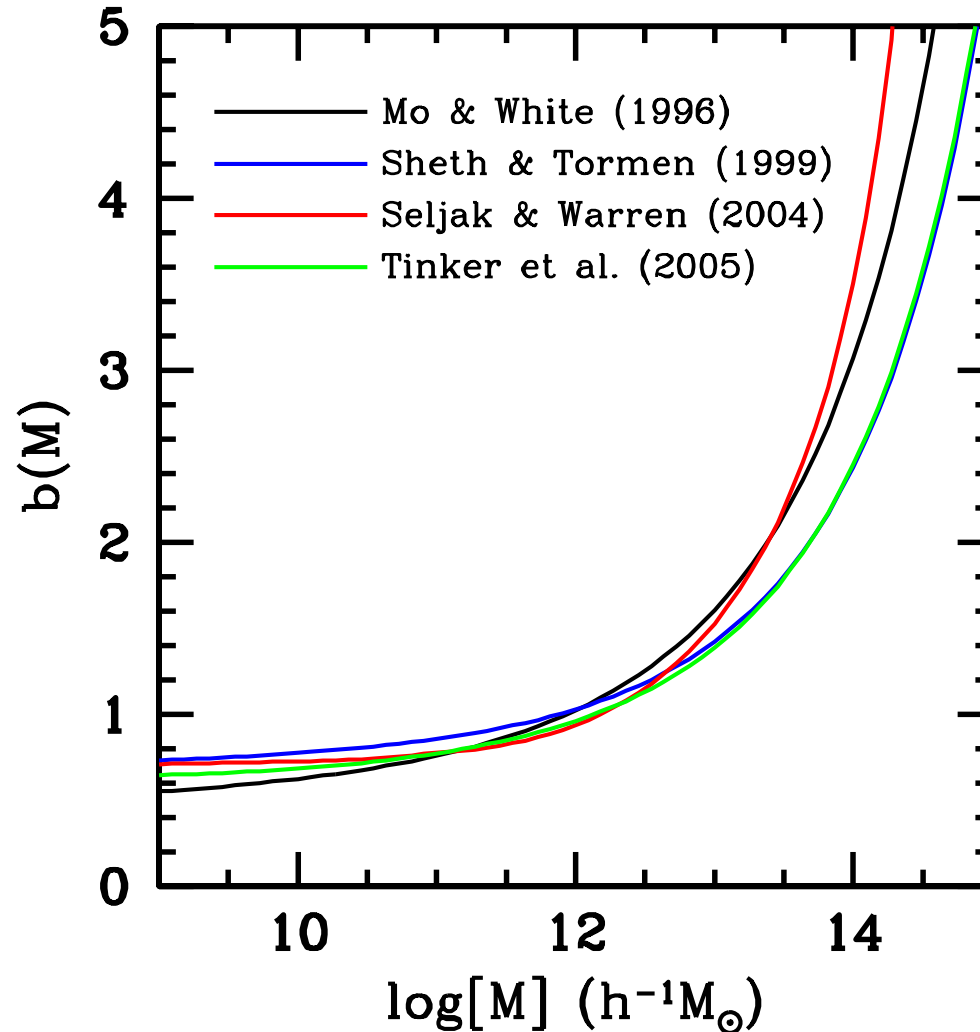
$$1 - \frac{1}{\delta_{\text{sc}}} < b(m) < 1 \quad \text{if } m < m^* \quad \text{(anti-biased)}$$

Note that there is an absolute minimum to the halo bias.

Halo-Halo correlation function:

$$\xi_{\text{hh}}(r) \equiv \langle \delta_{h_1} \delta_{h_2} \rangle = b(m_1)b(m_2)\langle \delta_1 \delta_2 \rangle = b(m_1)b(m_2)\xi(r)$$

Fine-Tuning of Halo Bias



Modifications to the derived halo bias have been made based on **ellipsoidal collapse corrections**, and calibrations against **numerical simulations**.

Currently, this is one of the main sources of uncertainty in the **halo model**.

Halo Occupation Statistics

How many galaxies, on average, per halo?

Halo Occupation Distribution: The **HOD** $P(N|M)$ specifies the probability that a halo of mass M contains N galaxies.

Of particular importance: first moment $\langle N \rangle_M = \sum_{N=0}^{\infty} N P(N|M)$

How are galaxies distributed (**spatially & kinematically**) within halo?

Central Galaxy : located at center of dark matter halo.

Satellite Galaxies: $n_{\text{sat}}(r) \propto \rho_{\text{dm}}(r) \iff \sigma_{\text{sat}}(r) = \sigma_{\text{dm}}(r)$

Supported by distribution of sub-haloes in N -body simulations

What are physical properties of galaxies (**luminosity, color, morphology**)

One needs separate **HOD** for each sub-class of galaxies...

Introduce **Conditional Luminosity Function**, $\Phi(L|M)$, which expresses average number of galaxies with luminosity L that reside in halo of mass M

The Conditional Luminosity Function

The CLF $\Phi(L|M)$ is the direct link between halo mass function $n(M)$ and the galaxy luminosity function $\Phi(L)$:

$$\Phi(L) = \int_0^\infty \Phi(L|M) n(M) dM$$

The CLF contains a lot of important information, such as:

- halo occupation **numbers** as function of luminosity:

$$N_M(L > L_1) = \int_{L_1}^\infty \Phi(L|M) dL$$

- The average relation between **light** and **mass**:

$$\langle L \rangle(M) = \int_0^\infty \Phi(L|M) L dL$$

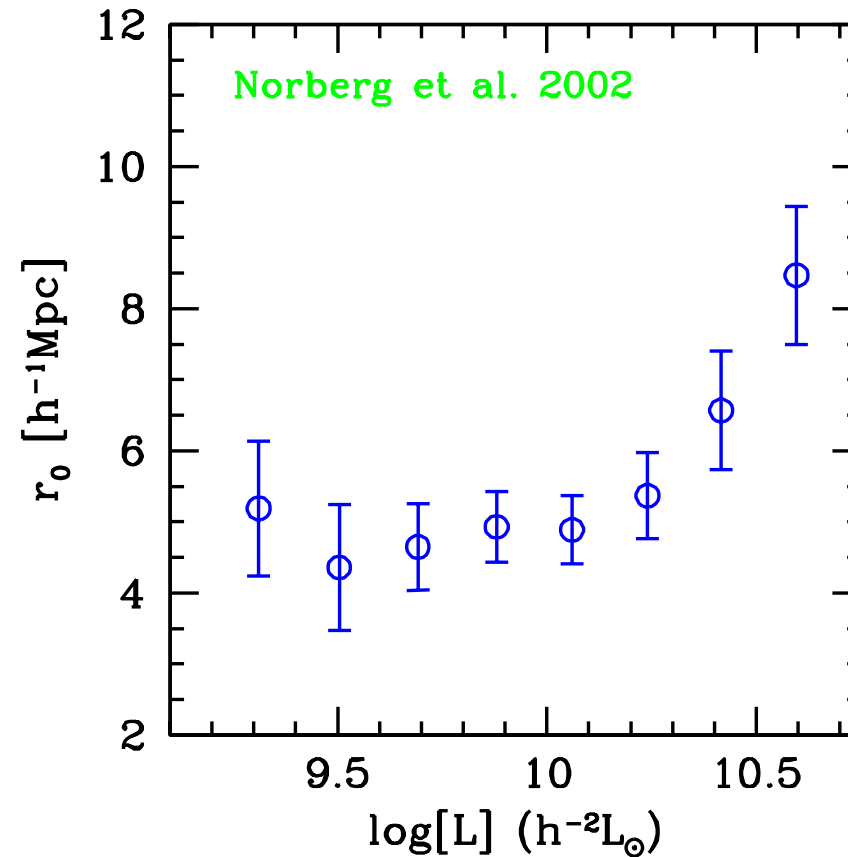
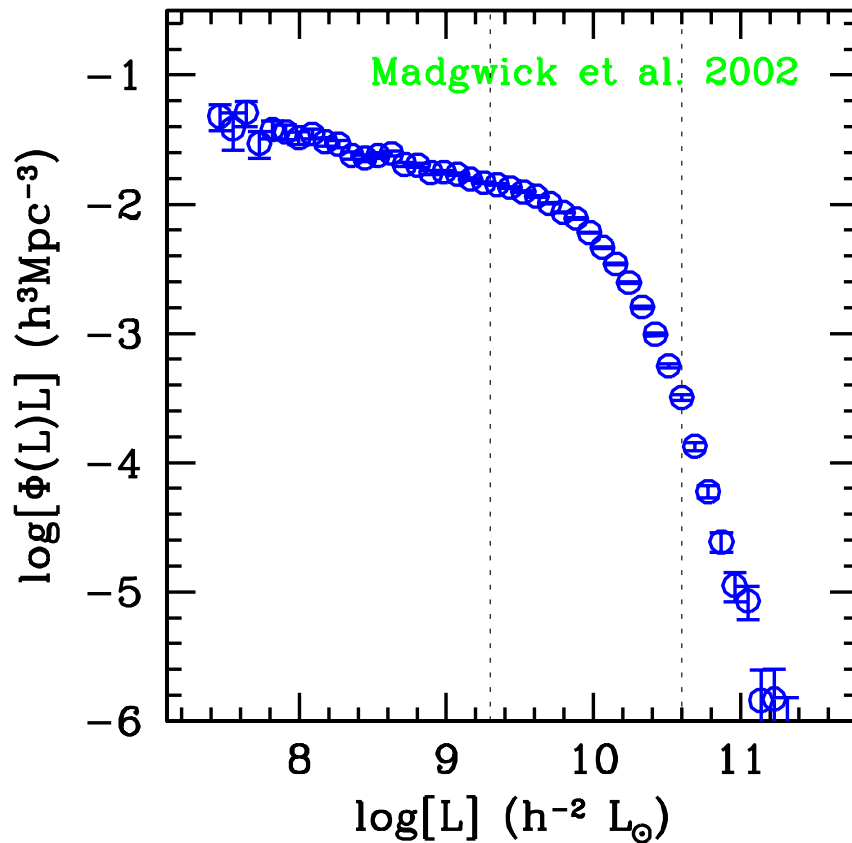
- The **bias** of galaxies as function of luminosity:

$$\xi_{\text{gg}}(r|L) = b^2(L) \xi_{\text{dm}}(r)$$

$$b(L) = \frac{1}{\Phi(L)} \int_0^\infty \Phi(L|M) b(M) n(M) dM$$

CLF is ideal statistical ‘tool’ to investigate Galaxy-Dark Matter Connection

Luminosity & Correlation Functions



- **2dFGRS:** More luminous galaxies are more strongly clustered.
- **Λ CDM:** More massive haloes are more strongly clustered.

More luminous galaxies reside in more massive haloes

REMINDER: Correlation length r_0 defined by $\xi(r_0) = 1$

The CLF Model

We **assume** that the CLF has the **Schechter** form:

$$\Phi(L|M)dL = \frac{\tilde{\Phi}^*}{\tilde{L}^*} \left(\frac{L}{\tilde{L}^*} \right)^{\tilde{\alpha}} \exp(-L/\tilde{L}^*) dL$$

Here $\tilde{\Phi}^*$, \tilde{L}^* and $\tilde{\alpha}$ all depend on M .

Use **Monte-Carlo Markov Chain** to find model that best fits $\Phi(L)$ and $r_0(L)$.

Predict $L_{\text{tot}}(M)$ and $L_c(M)$

(e.g., Yang et al. 2003; vdB et al. 2003a,b; vdB et al. 2005)

Alternative approach: split CLF in **central** and **satellite** components:

$$\begin{aligned}\Phi(L|M) &= \Phi_c(L|M) + \Phi_s(L|M) \\ \Phi_c(L|M) &= \frac{\tilde{\Phi}_c^*}{\sqrt{2\pi \ln(10)} L \sigma_c} \exp \left[-\frac{\log(L/\tilde{L}_c)^2}{2\sigma_c^2} \right] \\ \Phi_s(L|M) &= \tilde{\Phi}_s^* L^{\tilde{\alpha}} g(L - \tilde{L}_c)\end{aligned}$$

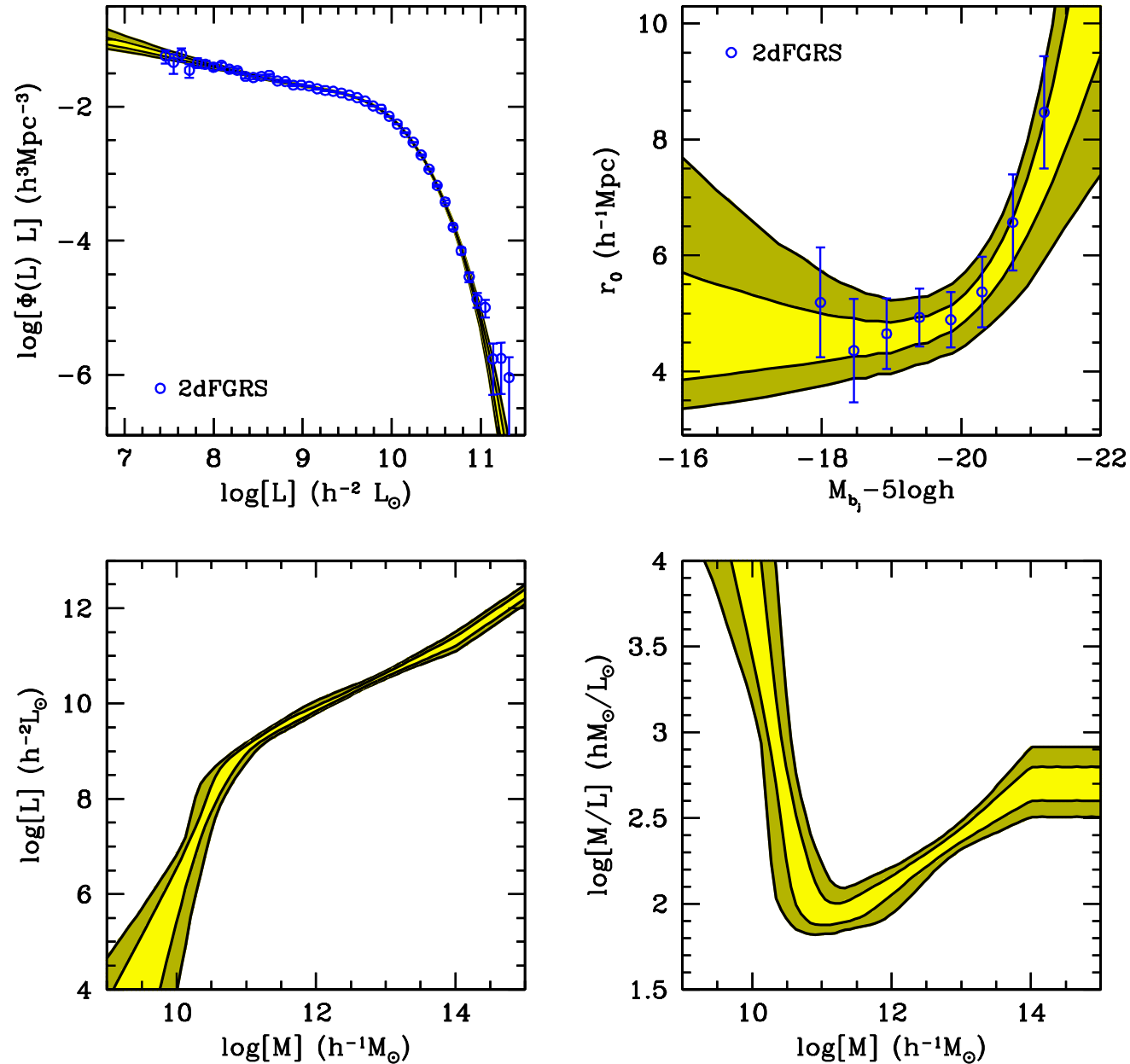
Here $\tilde{\Phi}_c^*$, $\tilde{\Phi}_s^*$, \tilde{L}_c and $\tilde{\alpha}$, all depend on M

Use direct observational constraints on $L_c(M)$, $L_{\text{tot}}(M)$ and $\alpha(M)$

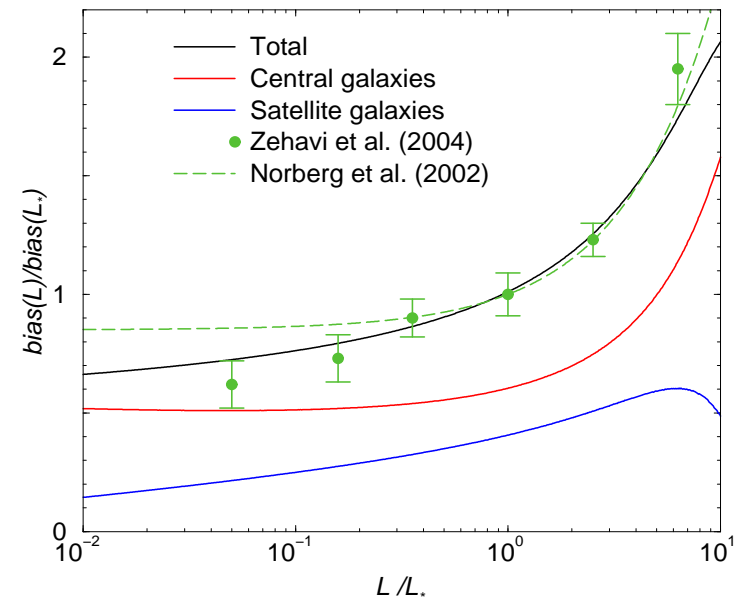
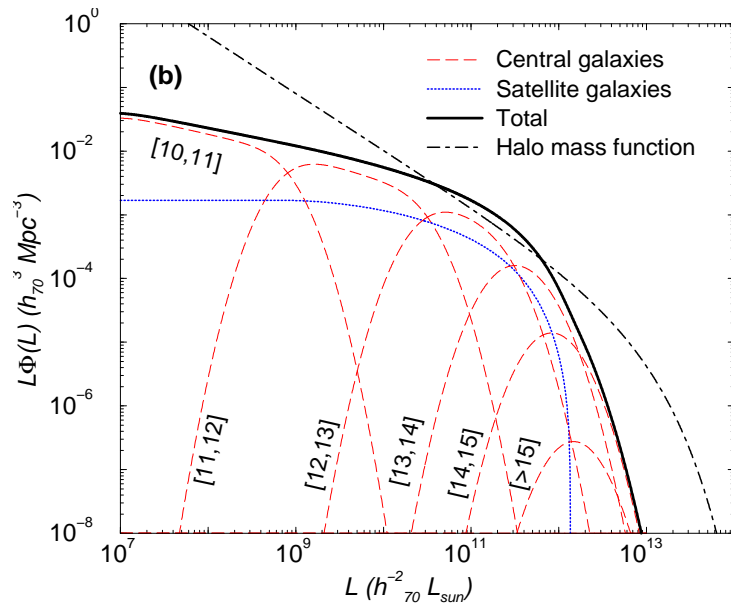
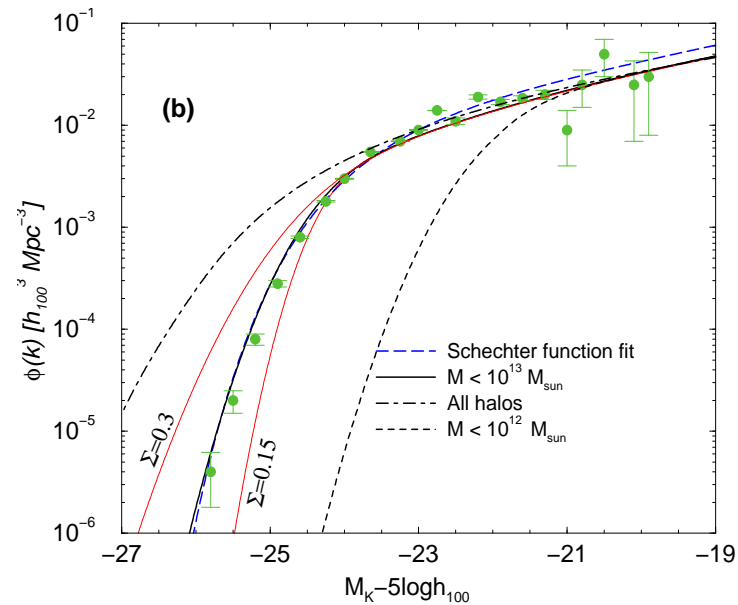
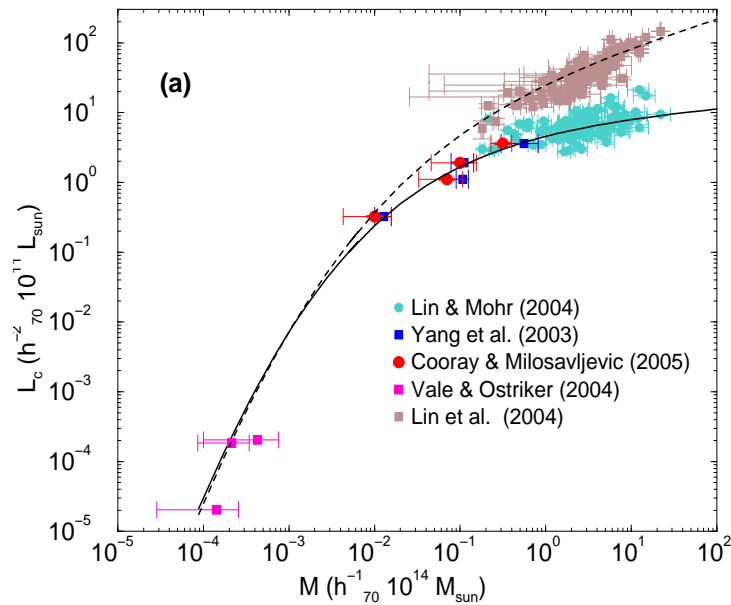
Predict $\Phi(L)$ and $r_0(L)$

(e.g., Cooray & Milosavljevic 2005; Cooray 2005a,b; Cooray 2006)

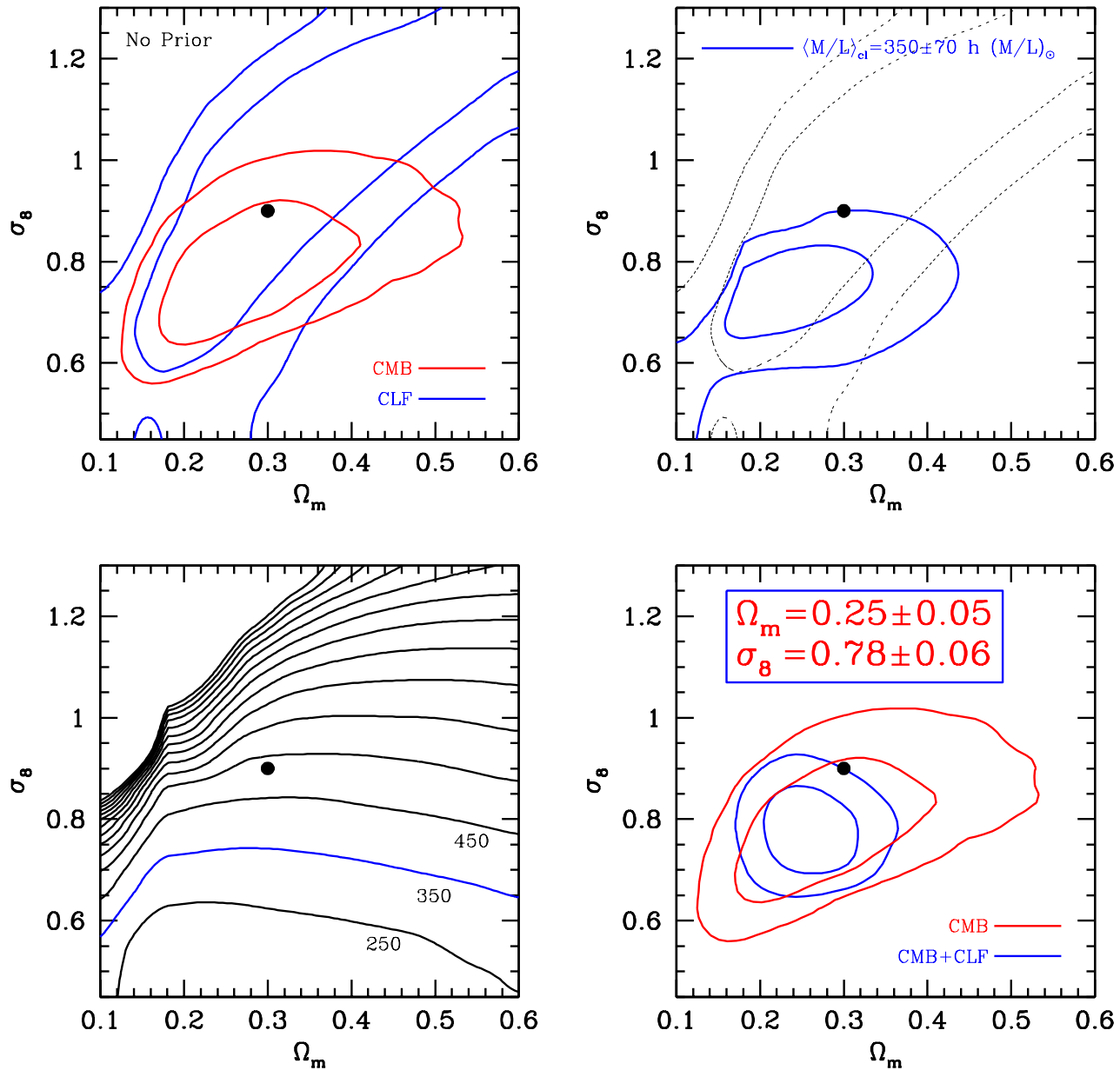
The Relation between Light & Mass



The Relation between Light & Mass



Cosmological Constraints

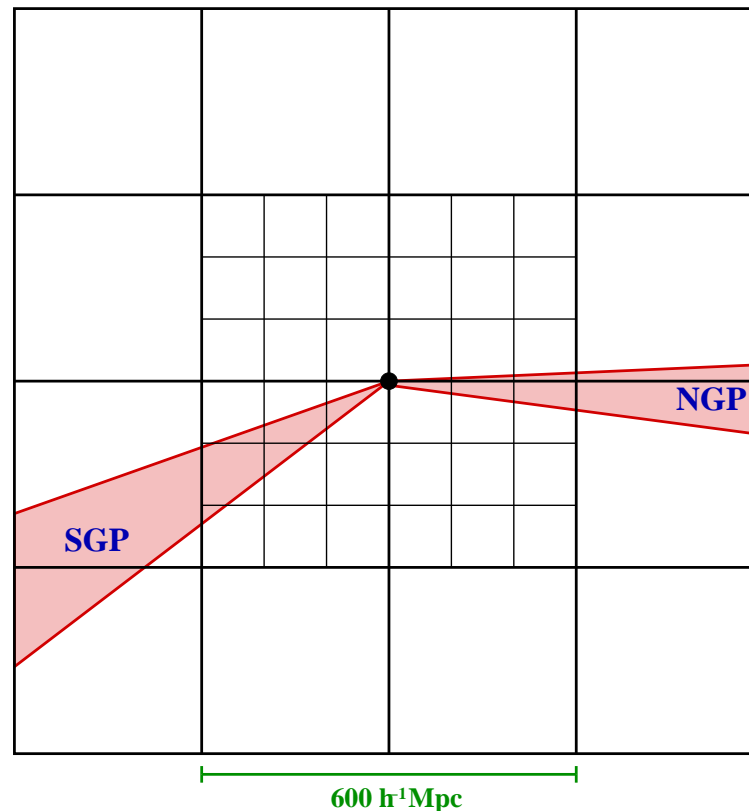


vdB, Mo & Yang, 2003, MNRAS, 345, 923

See also Tinker et al. 2005; Vale & Ostriker 2005

Constructing Mock Surveys

- Run **numerical simulations**: Λ CDM concordance cosmology (WMAP1)
 $L_{\text{box}} = 100h^{-1} \text{ Mpc}$ and $300h^{-1} \text{ Mpc}$ with 512^3 CDM particles each.
- Identify **dark matter haloes** with (**FOF** algorithm).
- **Populate haloes** with galaxies using **CLF**.
- Stack boxes to create **virtual universe** and mimic observations
(**magnitude limit, completeness, geometry, fiber collisions**)



HODs from Galaxy Groups

Halo Occupation Statistics can also be obtained **directly** from galaxy groups

Potential Problems: interlopers, (in)completeness, mass estimates

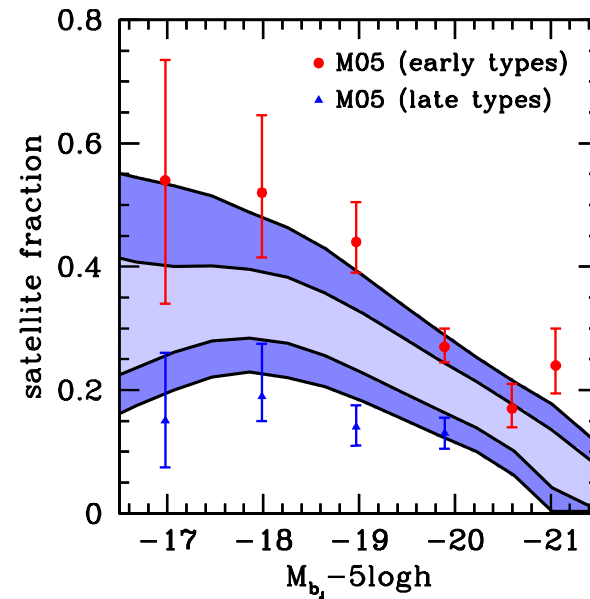
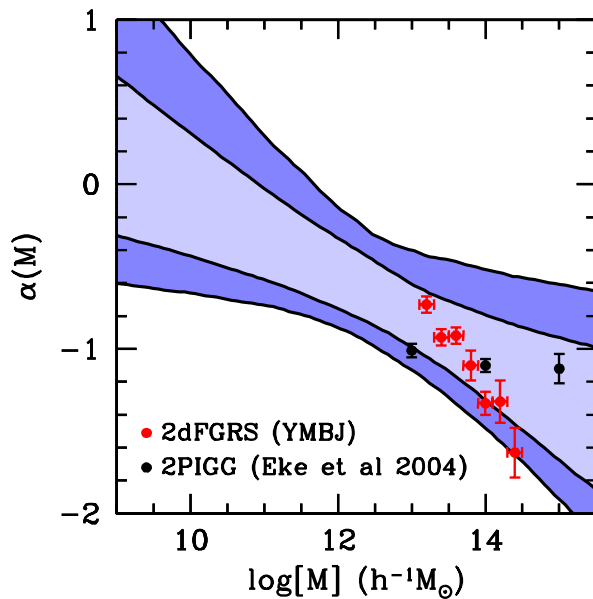
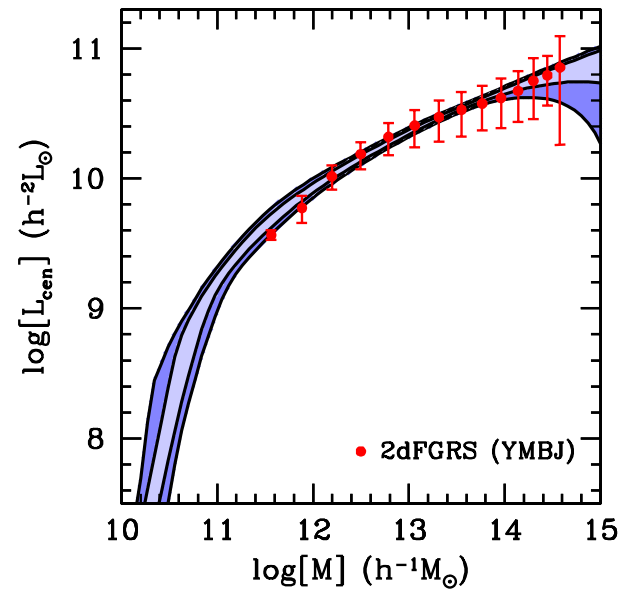
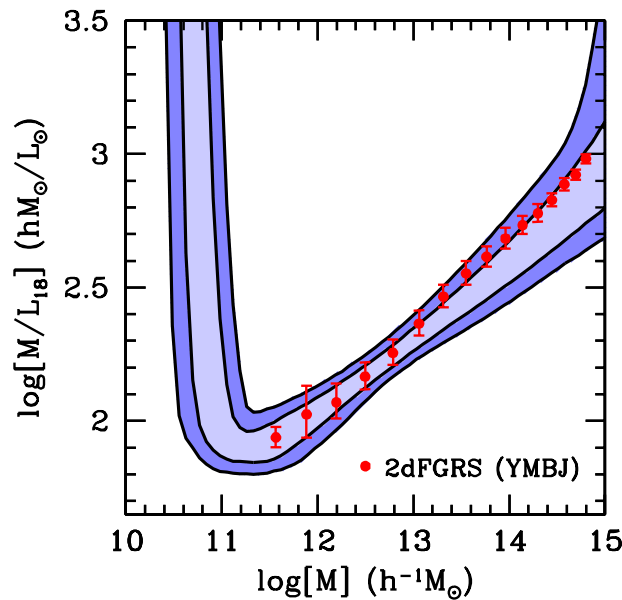
We developed new, iterative group finder, using an adaptive filter modeled after halo virial properties

Yang, Mo, vdB, Jing 2005, MNRAS, 356, 1293

- Calibrated & Optimized with **Mock Galaxy Redshift Surveys**
- Low **interloper** fraction ($\lesssim 20\%$).
- High **completeness** of members ($\gtrsim 90\%$).
- **Masses** estimated from group luminosities.
More accurate than using **velocity dispersion** of members.
- Can also detect “groups” with single member
 - ▷ Large dynamic range ($11.5 \lesssim \log[M] \lesssim 15$).

Group finder has been applied to both the **2dFGRS** (completed survey) and to the **SDSS** (DR2, NYU-VAGC; Blanton et al. 2005)

The Relation between Light & Mass



YMBJ = Yang, Mo, vdB & Jing, 2005

M05 = Mandelbaum et al. 2005

vdB et al. 2006, in prep.

Galaxy Ecology

Many studies have investigated relation between various **galaxy properties** (morphology / SFR / colour) and **environment**

(e.g., Dressler 1980; Balogh et al. 2004; Goto et al. 2003; Hogg et al. 2004)

Environment estimated using **galaxy overdensity** (projected) to n^{th} nearest neighbour, Σ_n or using fixed, metric aperture, Σ_R .

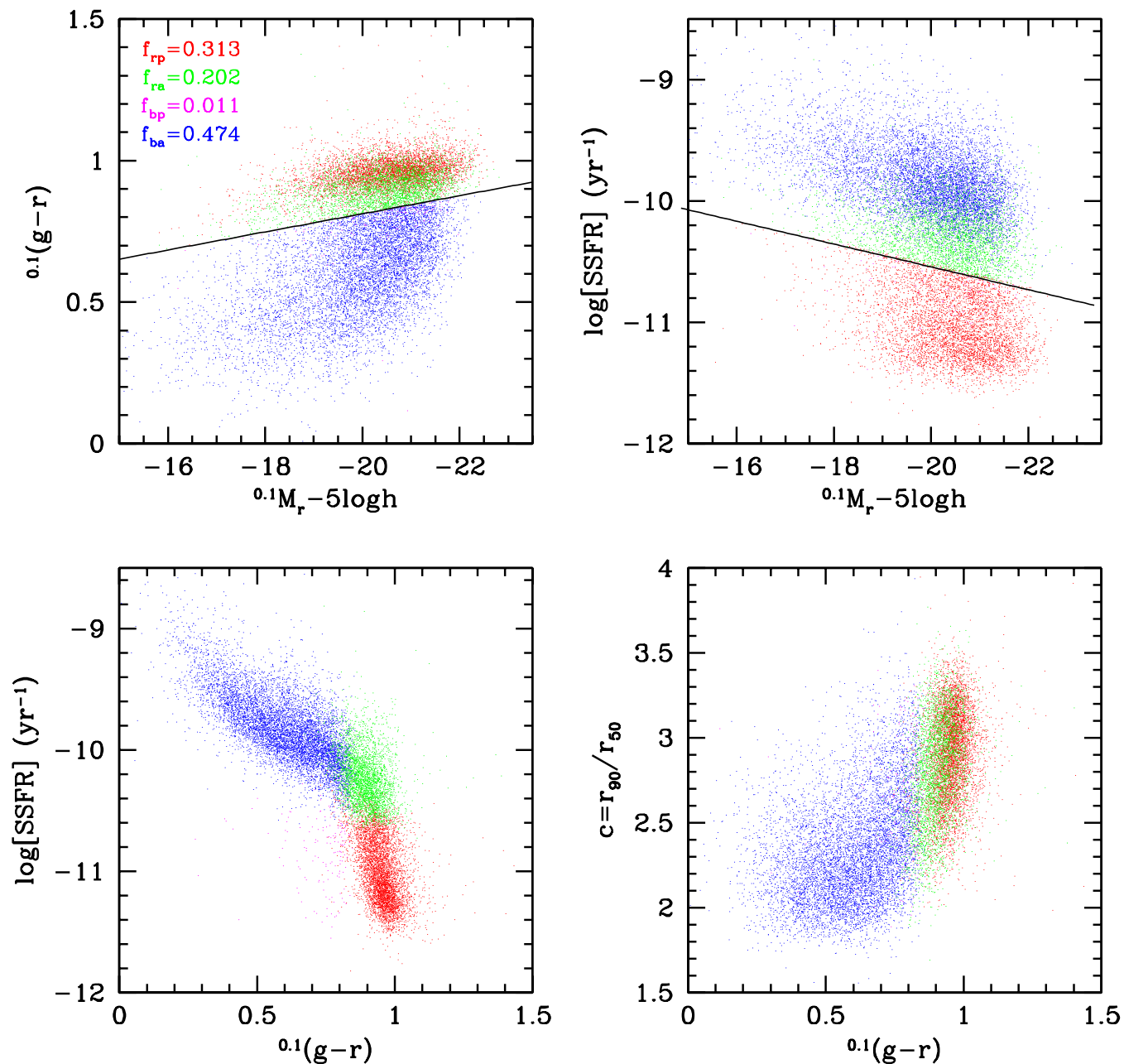
- Fraction of early types **increases** with density
- There is a **characteristic density** (\sim group-scale) below which environment dependence vanishes
- Groups and Clusters reveal **radial dependence**:
late type fraction increases with radius
- No radial dependence in groups with $M \lesssim 10^{13.5} h^{-1} M_{\odot}$

Danger: Physical meaning of Σ_n and Σ_R depends on environment.

Physically more meaningful to investigate **halo mass dependence** of galaxy properties. This requires **galaxy group catalogues**.

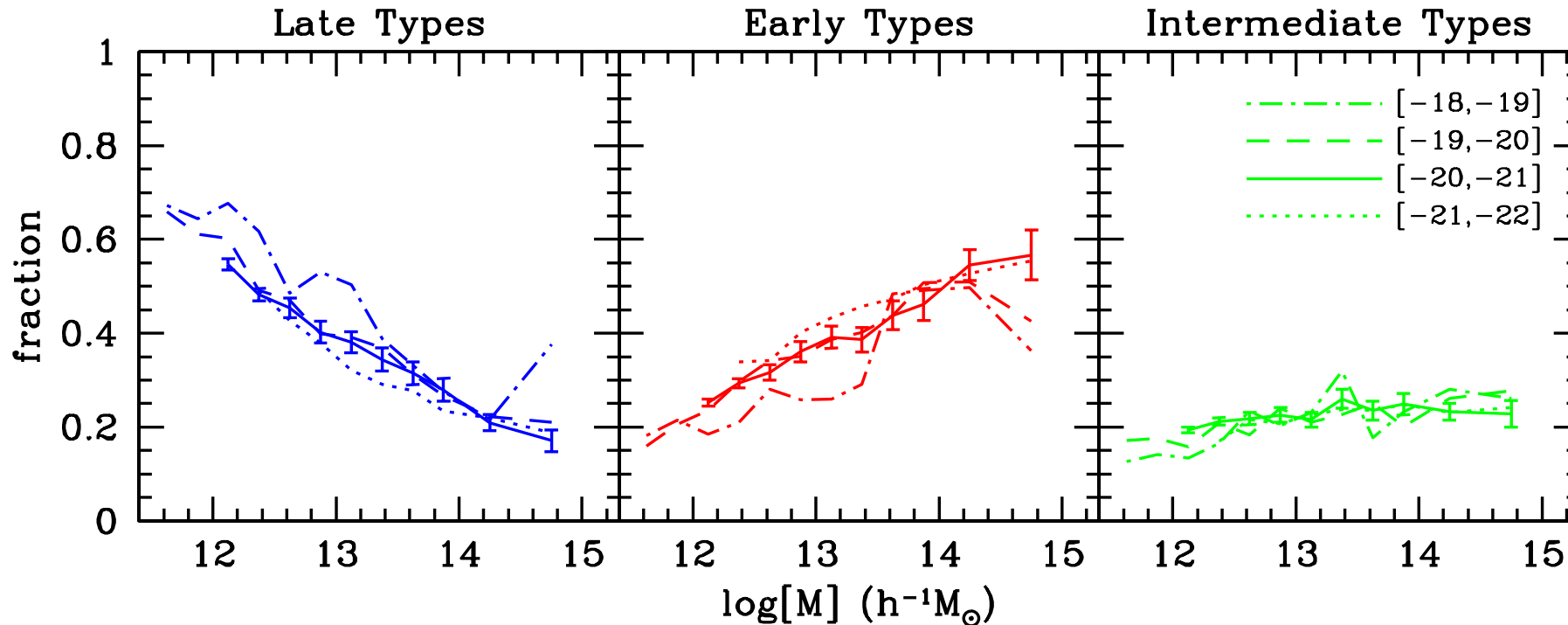
Important: Separate **luminosity dependence** from **halo mass dependence**.

Defining Galaxy Types



Data from NYU-VAGC (Blanton et al. 2005): SSFRs from Kauffmann et al. (2003) and Brinchmann et al. (2004)

Halo Mass Dependence



The fractions of **early** and **late** type galaxies depend strongly on halo mass.

At fixed halo mass, there is virtually **no luminosity dependence**.

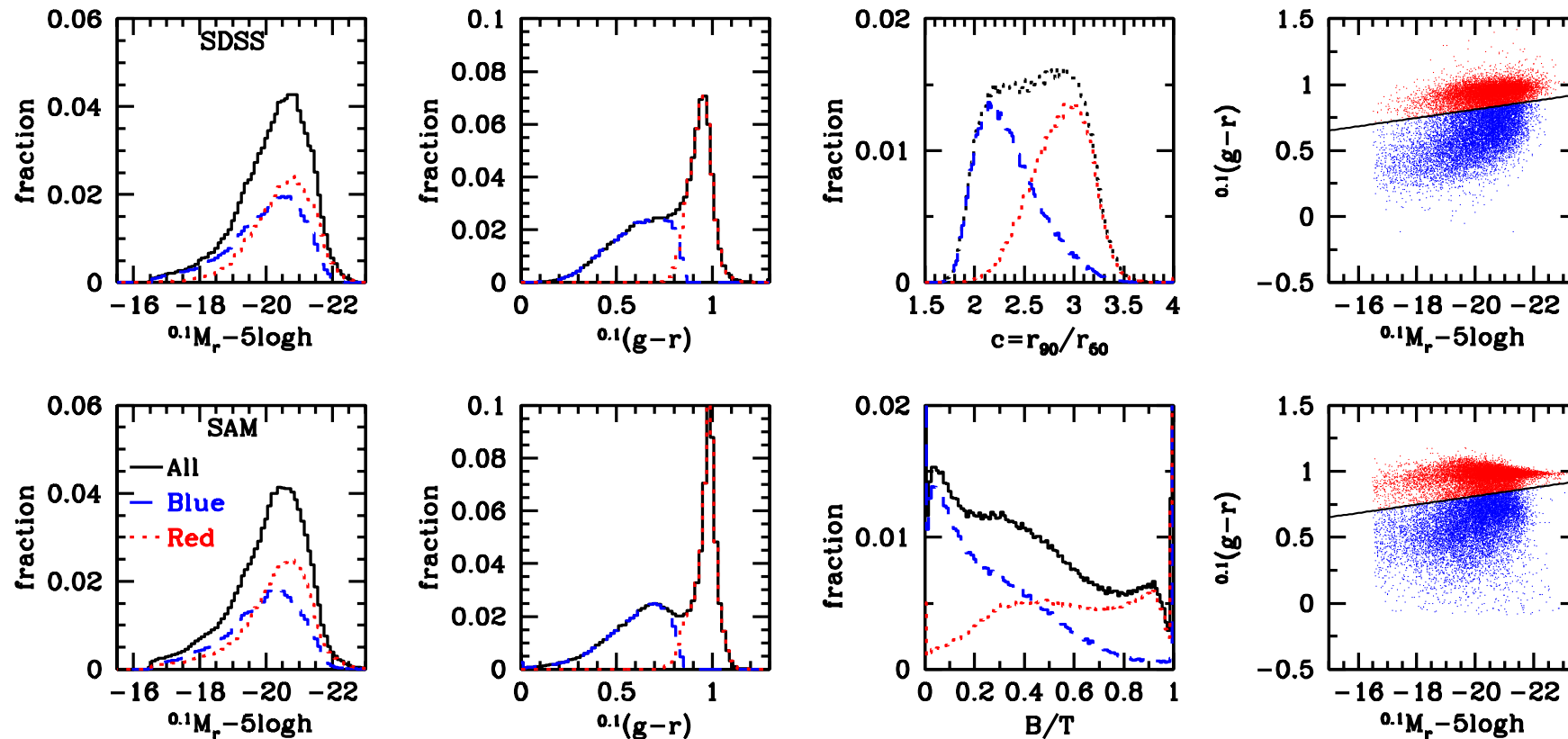
The mass dependence is smooth: there is **no characteristic mass scale**

The **intermediate** type fraction is independent of luminosity and mass.

(Weinmann, vdB, Yang & Mo, 2006)

Comparison with Semi-Analytical Model

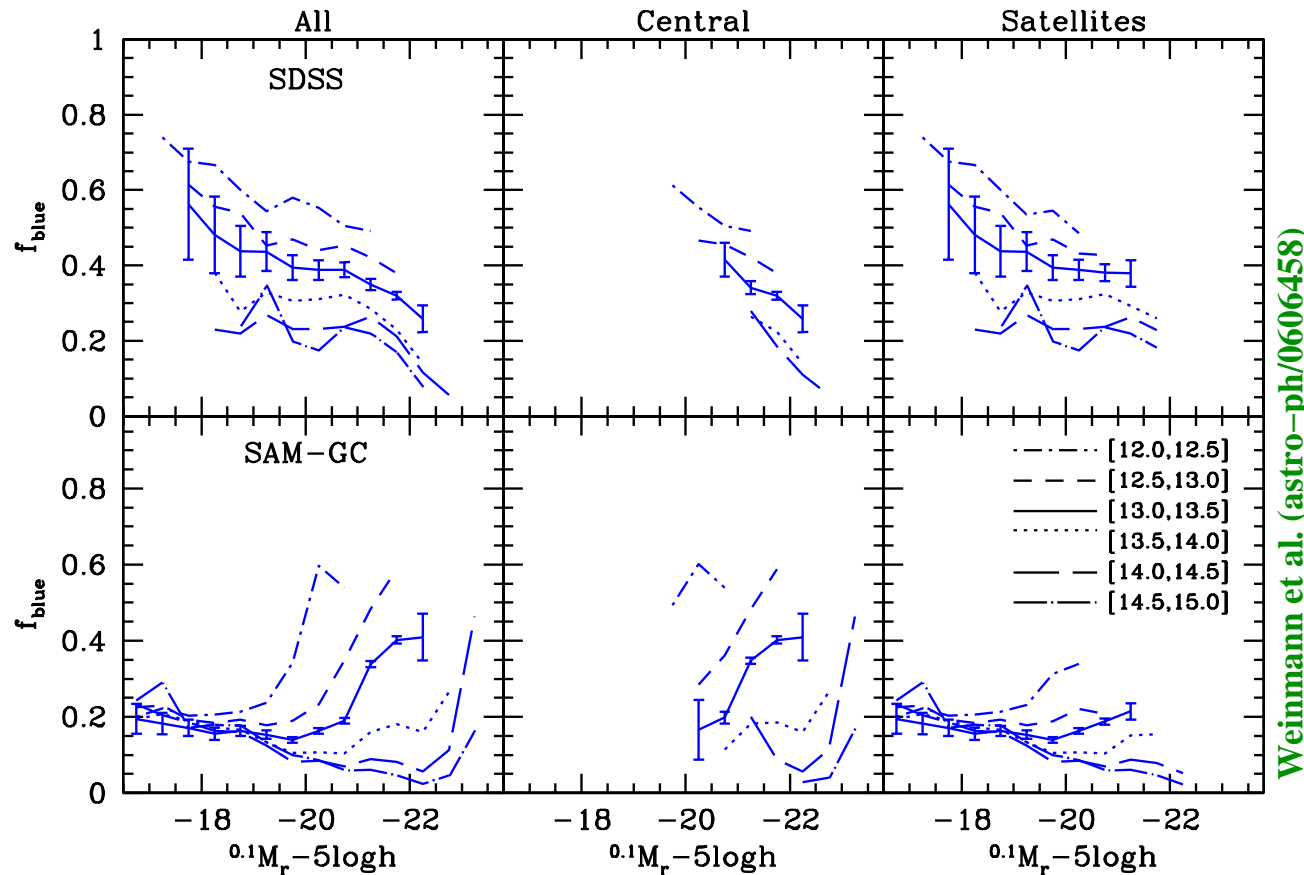
Comparison of **Group Occupation Statistics** with **Semi-Analytical Model** of **Croton et al. 2006**. Includes 'radio-mode' AGN feedback.



- **SAM** matches **global statistics** of **SDSS**
- Luminosity function, bimodal color distribution, and overall blue fraction
- But what about statistics as function of halo mass?

Constraining Star Formation Truncation

To allow for fair comparison, we run our Group Finder over **SAM**.



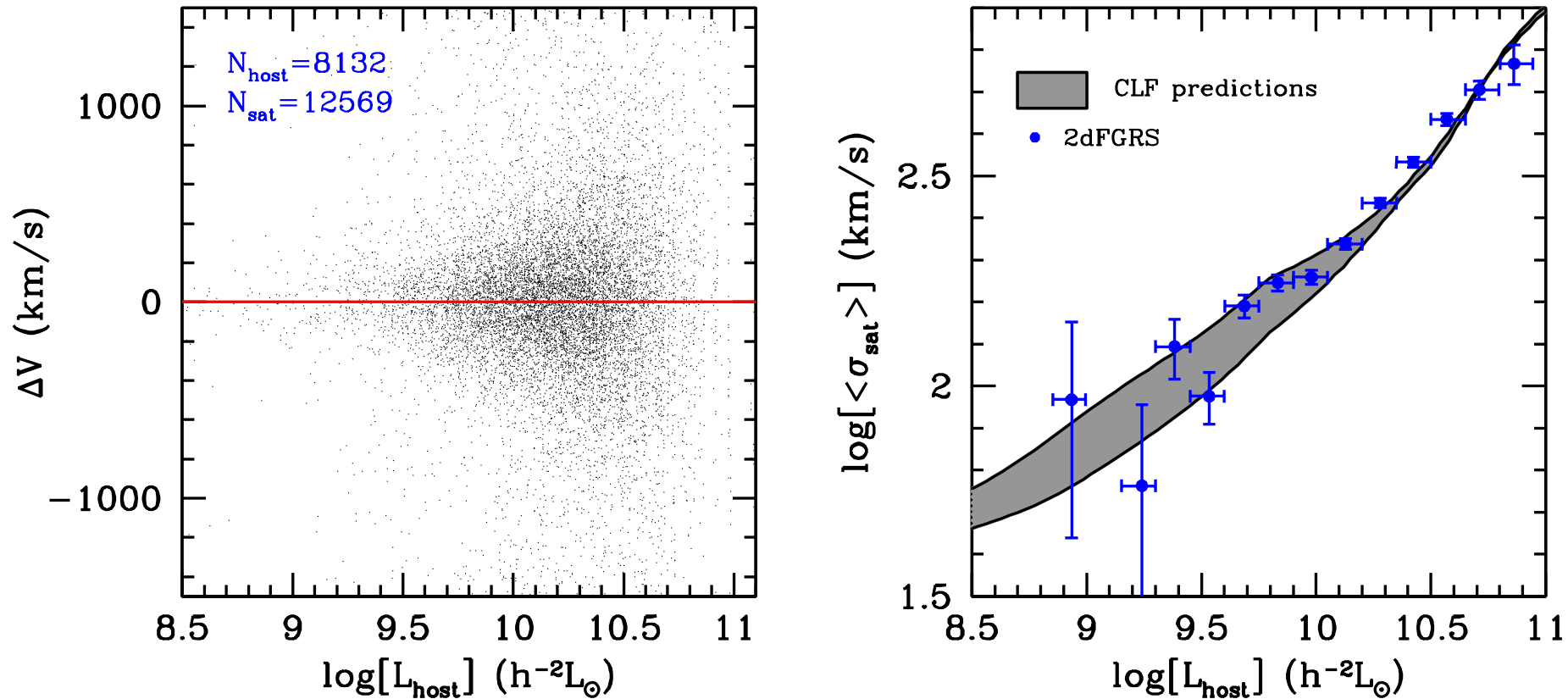
Weinmann et al. (astro-ph/0606458)

Satellites: red fraction too large: \triangleright **strangulation** too efficient as modelled

Centrals: $f_{\text{blue}}(L|M)$ wrong: \triangleright **AGN feedback/dust modelling** wrong

$f_{\text{blue}}(L, M)$ useful to constrain SF truncation mechanism

Satellite Kinematics in the 2dFGRS

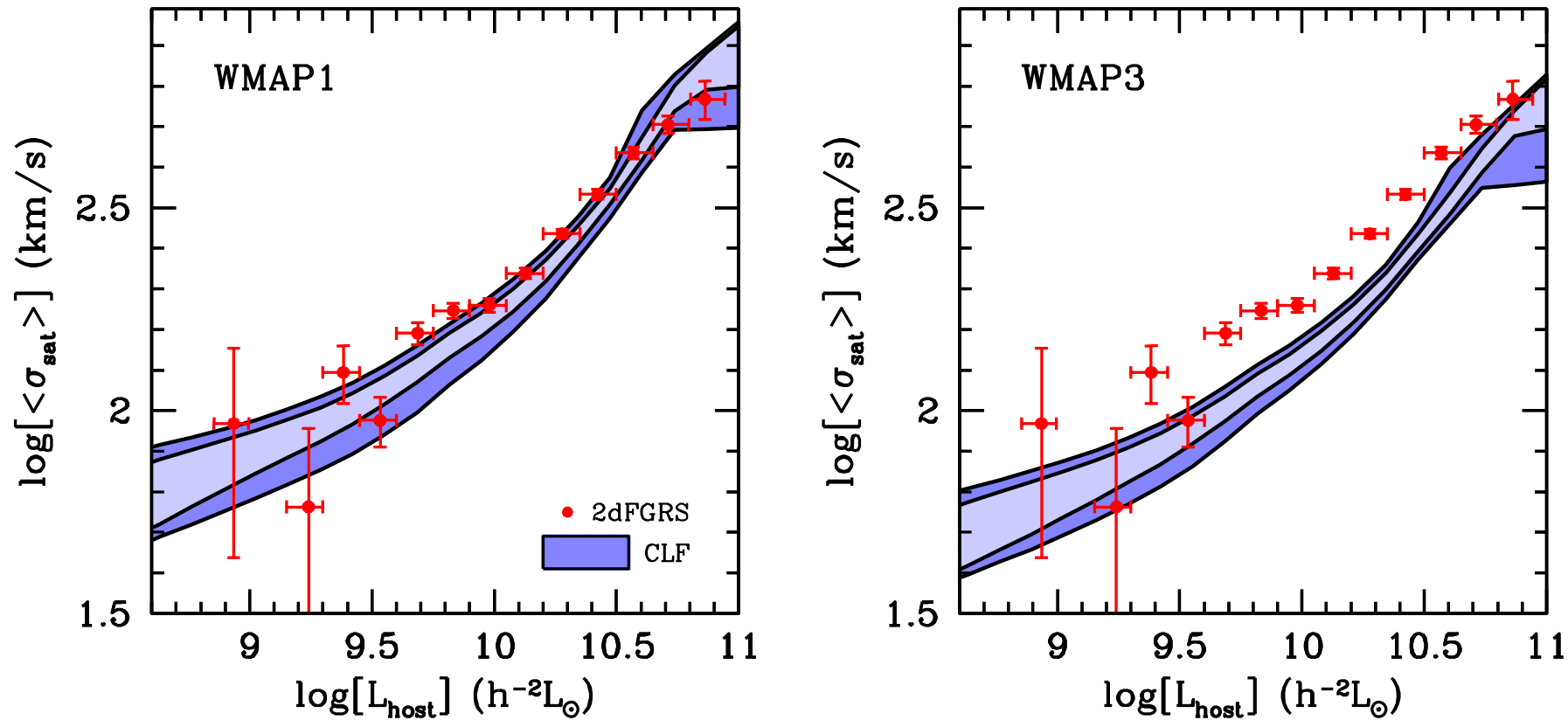


- Mocks are used to **optimize** host-satellite selection criteria
- Using an **iterative, adaptive** selection criterion **minimizes** interlopers
- Application to **2dFGRS** yields 12569 satellites & 8132 hosts
- Independent **dynamical evidence** to support **WMAP1-CLF** results

vdB, Norberg, Mo & Yang, 2004, MNRAS, 352, 1302

vdB, Yang, Mo & Norberg, 2005, MNRAS, 356, 1233

Problems for the WMAP3 Cosmology?



- In **WMAP3** cosmology, haloes have lower mass-to-light ratios and are less concentrated.
- **WMAP3-CLF** underpredicts satellite velocity dispersions by $\sim 30\%$
- But, $L_{\text{cen}}(M)$ in good agreement with group-data....
- Central galaxies do **not** reside at rest at center of halo.

Brightest Halo Galaxies

Paradigm: Brightest Galaxy in halo resides at rest at center

In order to test this **Central Galaxy Paradigm**, we compare the velocity of central galaxy to the average velocity of the satellites. Define

$$\mathcal{R} = \frac{N_s(v_c - \bar{v}_s)}{\hat{\sigma}_s}$$

with $\bar{v}_s = \frac{1}{N_s} \sum_{i=1}^{N_s} v_i$ and $\hat{\sigma}_s = \sqrt{\frac{1}{N_s - 1} \sum_{i=1}^{N_s} (v_i - \bar{v}_s)^2}$.

If **Central Galaxy Paradigm** is correct, $P(\mathcal{R})$ follows a Student t-distribution with $N_s - 1$ degrees of freedom

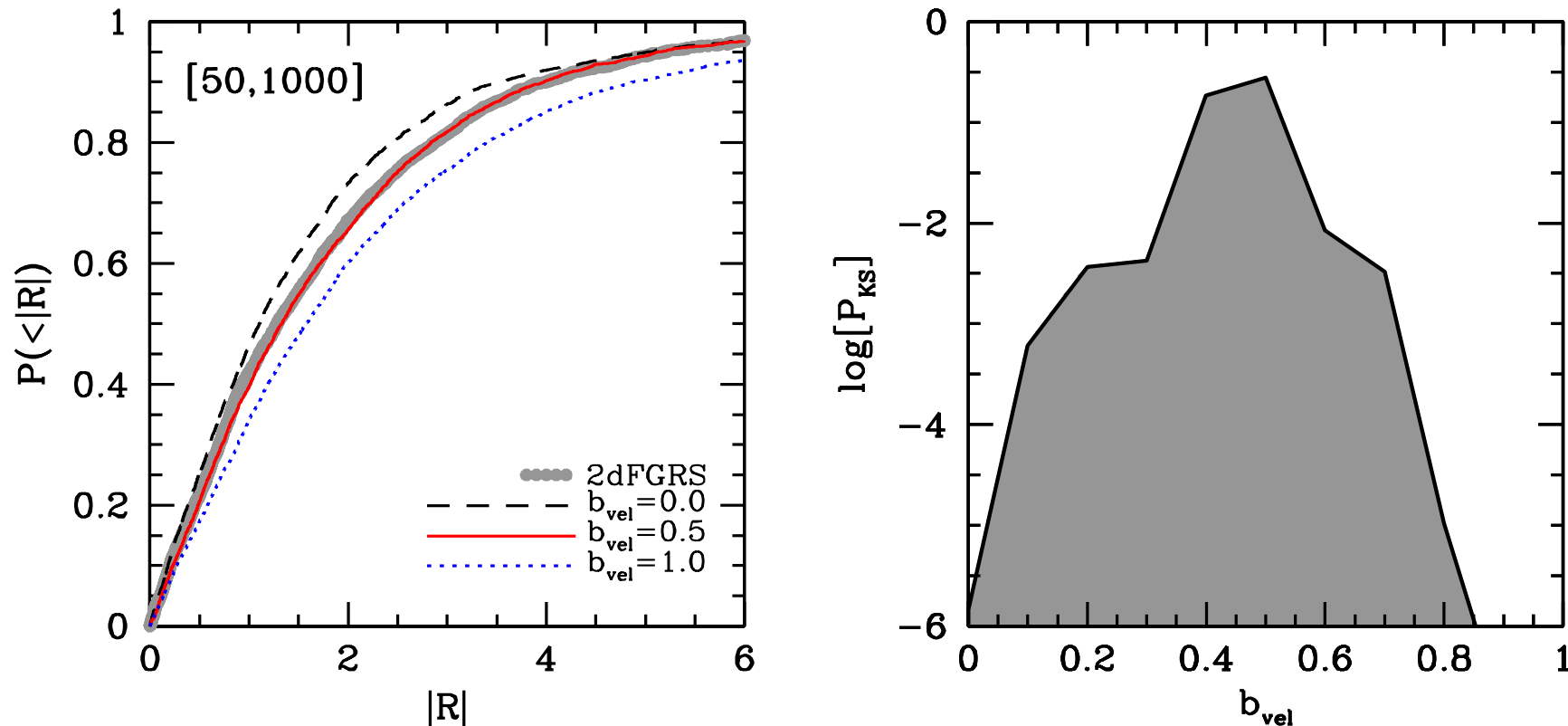
IMPORTANT: Applicability of this \mathcal{R} -test depends strongly on ability to find those galaxies that belong to same halo.

PROBLEM: Interlopers and incompleteness effects

SOLUTION: Use halo-based group finder and mock galaxy redshift surveys

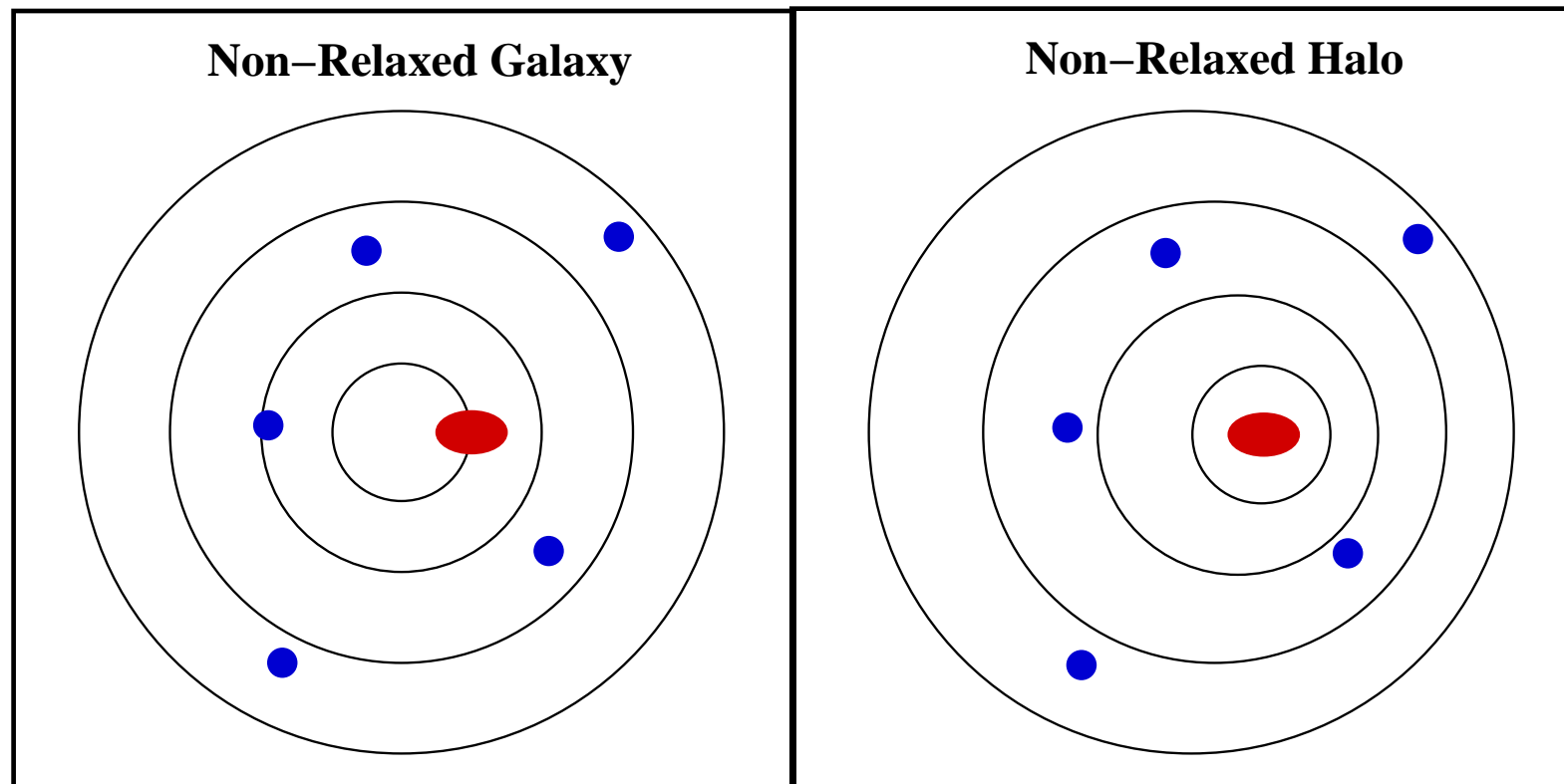
DATA: Both **2dFGRS** (Final Data Release) and **SDSS** (DR2, NYU-VAGC)

Evidence against Central Galaxy Paradigm



- We construct **ten MGRSs**, that only differ in the **velocity bias** (b_{vel}) of the brightest halo galaxy
- The $P(\mathcal{R})$ of **2dFGRS** is best reproduced by **MGRS** with $b_{\text{vel}} = 0.5$
- The null-hypothesis of the **Central Galaxy Paradigm** is ruled out at strong confidence: $P_{\text{KS}} = 1.5 \times 10^{-6}$
- Best-fit value of $b_{\text{vel}} = 0.5$ suggests that **specific kinetic energy** of central galaxies is $\sim 25\%$ of that of satellites

Implications



- Brightest halo galaxy either **oscillates** in **relaxed** halo, or resides at **potential minimum** of **non-relaxed** halo.
- Strong gravitational lensing (**external shear?**)
- Distortions in disk galaxies (**lopsidedness & bars**)
- Satellite kinematics $\sigma_{\text{sat}} = \sqrt{1 + b_{\text{vel}}} \sigma_{\text{dm}}$
with $b_{\text{vel}} = \langle |v_{\text{cen}}|^2 \rangle / \sigma_{\text{sat}}^2$

Large Scale Structure: Theory

Galaxy redshift surveys yield $\xi(r_p, \pi)$ with r_p and π the pair separations perpendicular and parallel to the line-of-sight.

redshift space CF: $\xi(s)$ with $s = \sqrt{r_p^2 + \pi^2}$

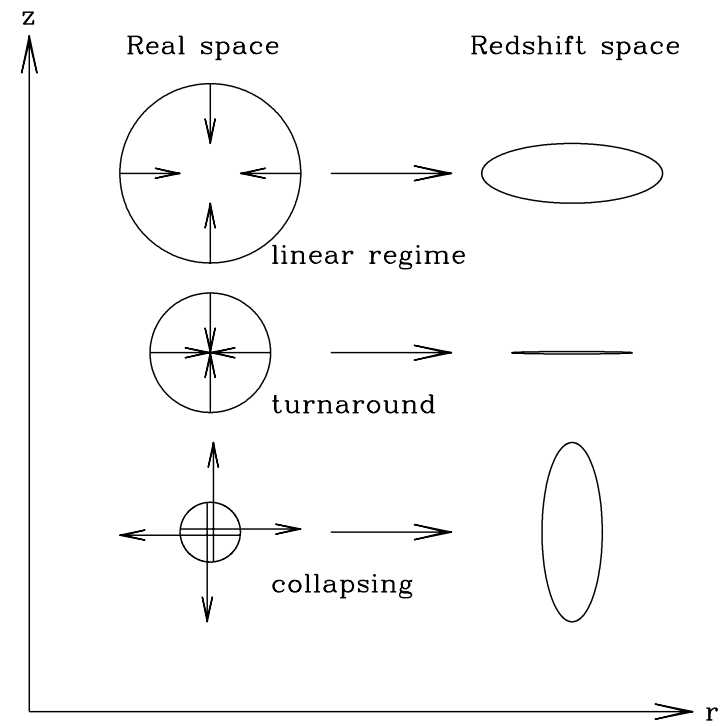
$$\text{projected CF: } w_p(r_p) = \int_{-\infty}^{\infty} \xi(r_p, \pi) d\pi = 2 \int_{r_p}^{\infty} \xi(r) \frac{r dr}{\sqrt{r^2 - r_p^2}}$$

Peculiar velocities cause $\xi(r_p, \pi)$ to be anisotropic.

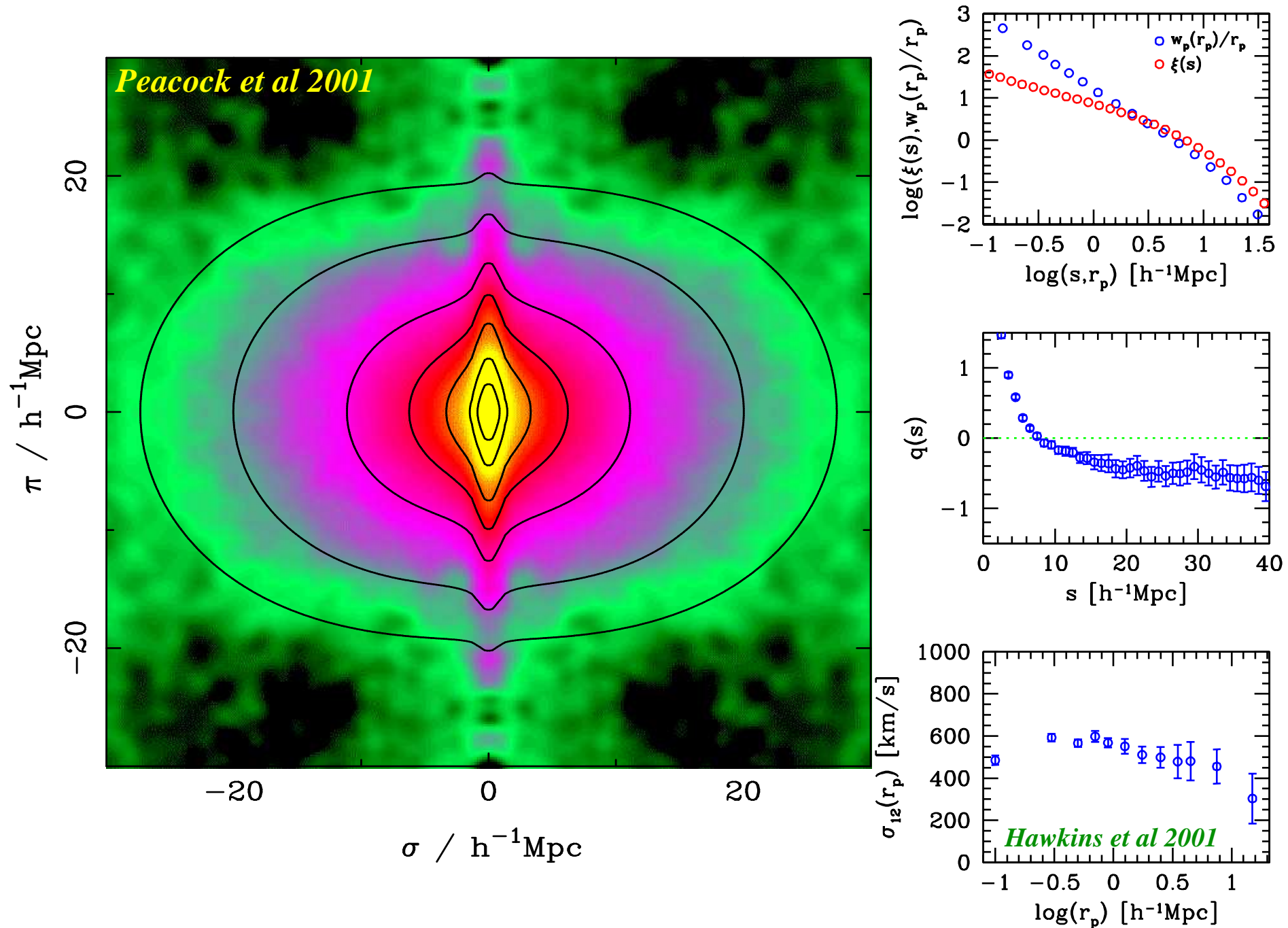
Consequently, $\xi(s) \neq \xi(r)$.

In particular, there are two effects:

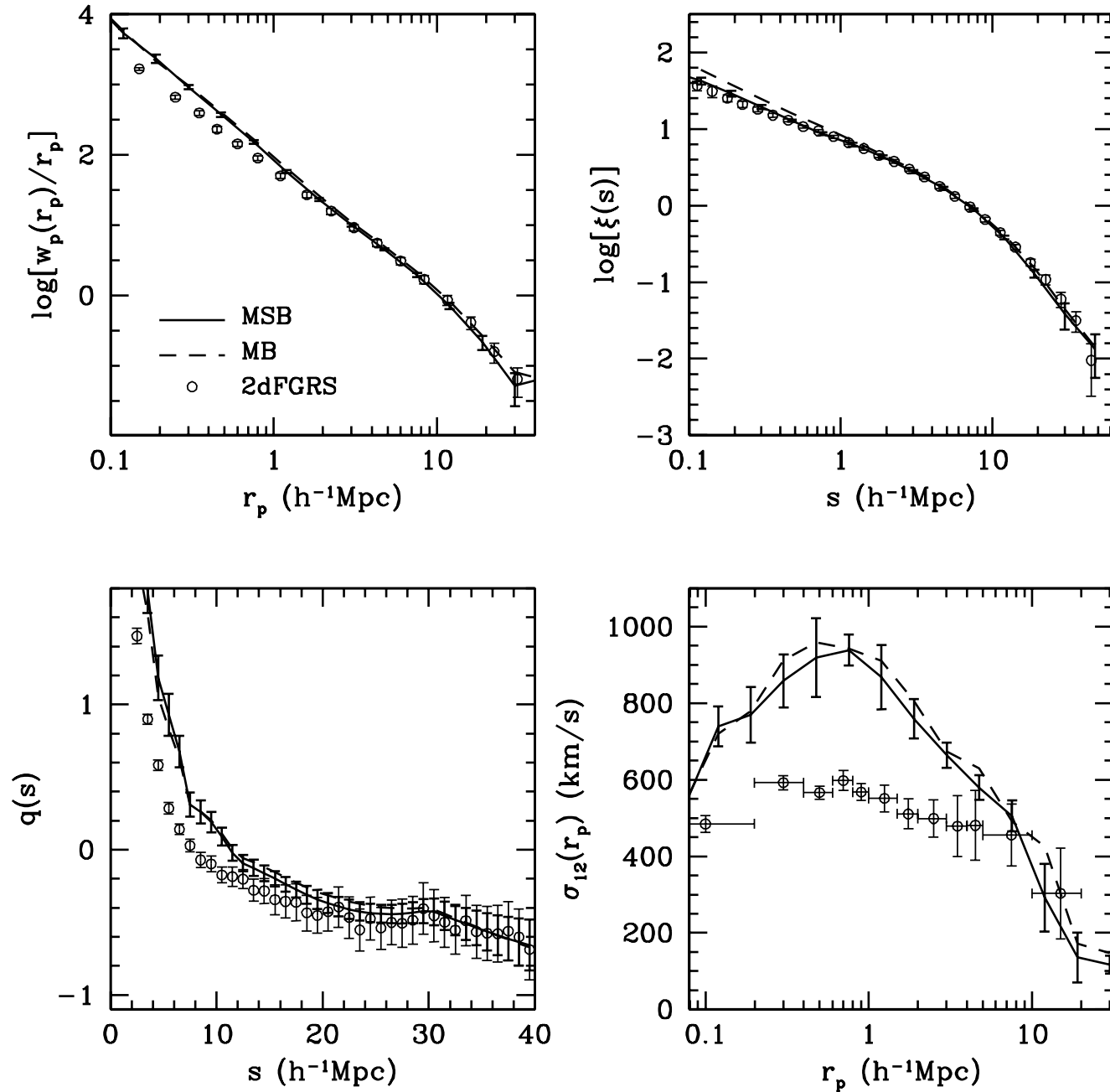
- **Large Scales:** Infall (“Kaiser Effect”)
- **Small Scales:** “Finger-of-God-effect”



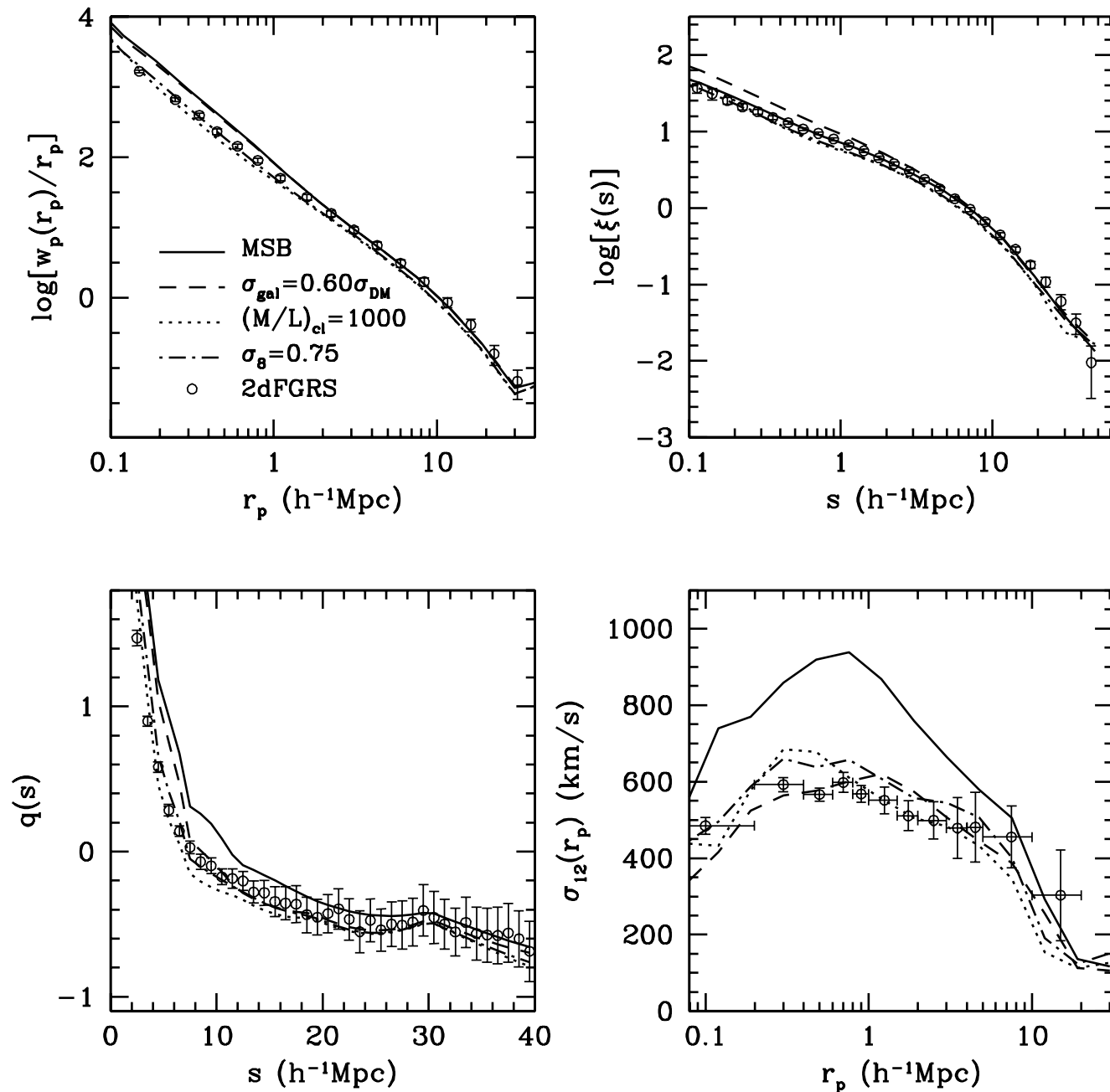
Large Scale Structure: The 2dFGRS



Mock versus 2dFGRS: round 1



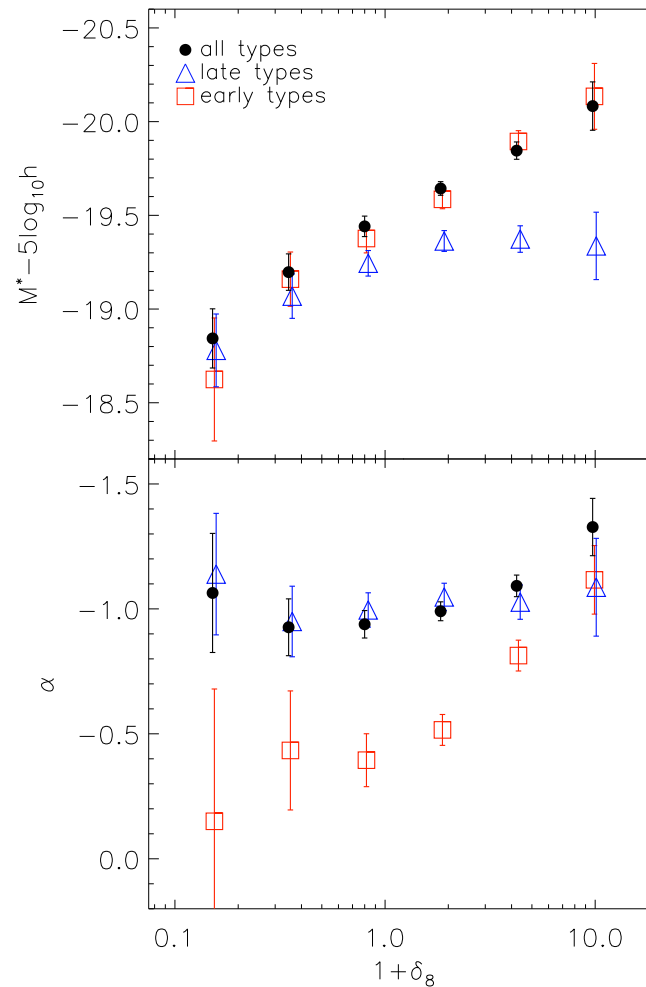
Mock versus 2dFGRS: round 2



Large-Scale Environment Dependence

Inherent to **CLF formalism** is assumption that L depends only on M .

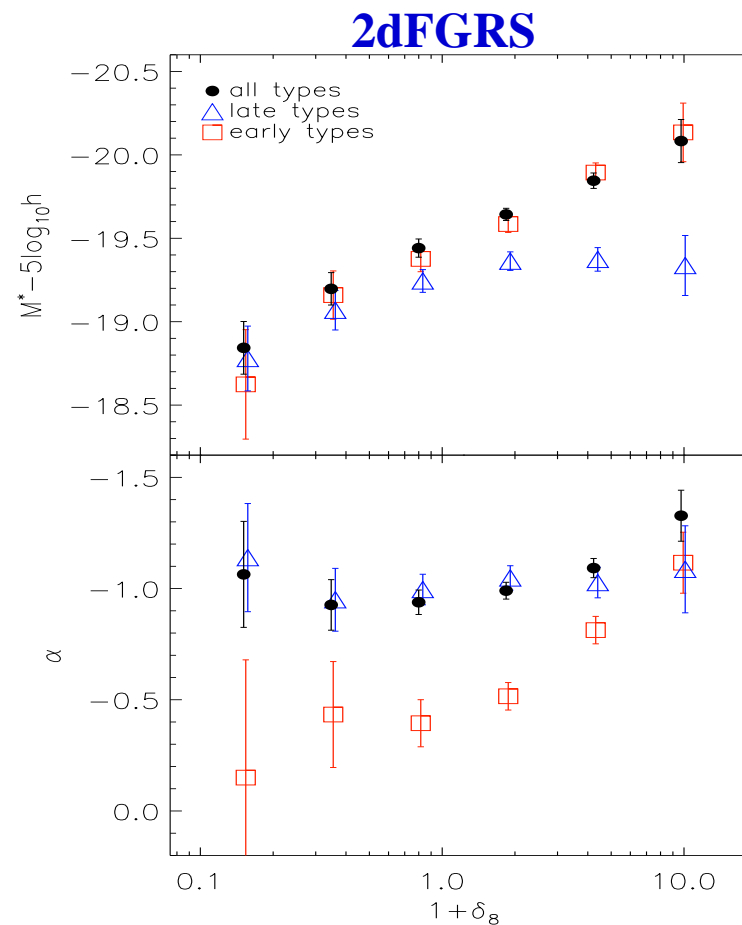
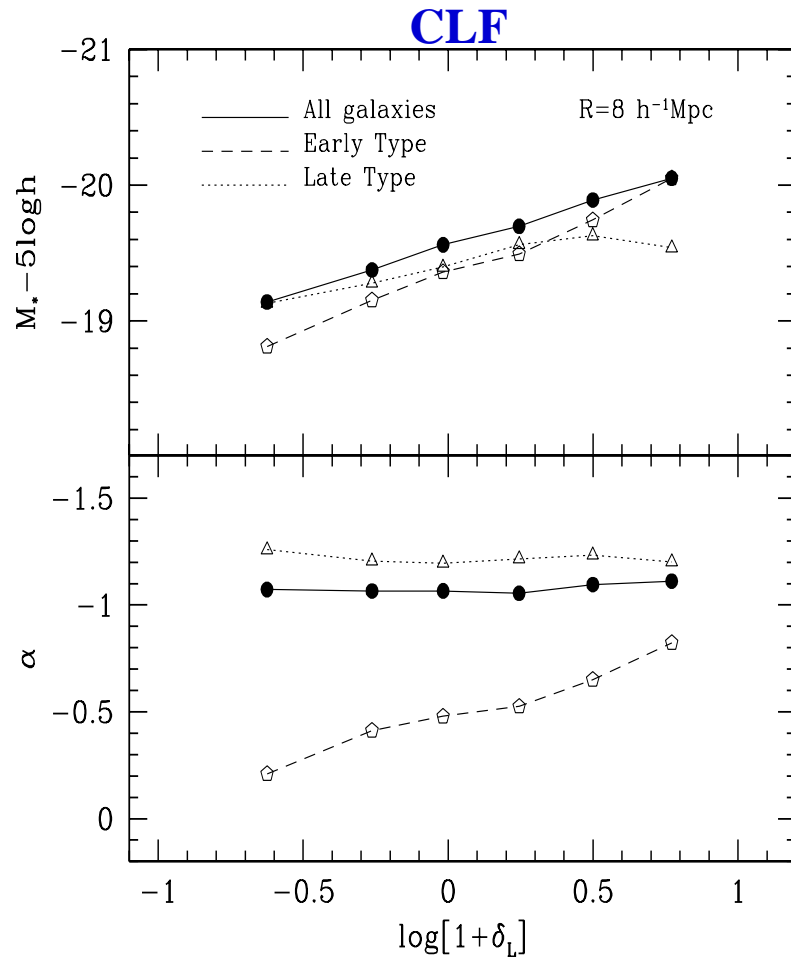
But $\Phi(L)$ has been shown to depend on **large scale environment**



Croton et al. 2005

Does this violate the implicit assumptions of the CLF formalism?

Large-Scale Environment Dependence



Populate haloes in N -body simulations with galaxies using $\Phi(L|M)$

Compute $\Phi(L)$ as function of environment and type as in Croton et al. (2005)

Because $n(M)$ depends on **environment**, we reproduce observed trend

There is no environment dependence, only halo-mass dependence

Theoretical Expectations

From the fact that

$$\delta_h(m) \equiv \frac{n(m|\delta)}{n(m)} - 1 = b(m)\delta$$

we obtain that

$$n(m|\delta) = [1 + b(m)\delta] n(m)$$

Since the halo bias $b(m)$ is an increasing function of halo mass, the abundance of more massive haloes is more strongly boosted in overdense regions than that of less massive haloes

In other words; **massive haloes live in overdense regions**

If more massive haloes host more luminous galaxies, we thus expect that the luminosity function of galaxies also depends on environment

Conclusions

Galaxy Bias = Halo Bias + Halo Occupation Statistics

Halo Occupation Statistics can be modeled & constrained using:

- Halo Occupation Distribution (HOD) $P(N|M)$
- Conditional Luminosity Function (CLF) $\Phi(L|M)$

or it can be ‘measured’ directly using **galaxy groups**

Halo Model and/or **Halo Occupation Statistics** can:

- Constrain Cosmological Parameters
- Constrain Galaxy Formation

In the near **future** we will be able to

- Constrain galaxy bias as function of redshift
- Obtain independent constraints from galaxy-galaxy lensing

Correlation Functions

Define the dimensionless density perturbation field: $\delta(\vec{x}) = \frac{\rho(\vec{x}) - \bar{\rho}}{\bar{\rho}}$

For a **Gaussian random field**, the **one-point probability function** is:

$$P(\delta) d\delta = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{\delta^2}{2\sigma^2}\right] d\delta$$

$$\langle \delta \rangle = \int \delta P(\delta) d\delta = 0$$

$$\langle \delta^2 \rangle = \int \delta^2 P(\delta) d\delta = \sigma^2$$

Define **n -point probability function**: $P_n(\delta_1, \delta_2, \dots, \delta_n) d\delta_1 d\delta_2 \dots d\delta_n$

Gravity induces **correlations** between δ_i so that

$$P_n(\delta_1, \delta_2, \dots, \delta_n) \neq \prod_{i=1}^n P(\delta_i)$$

Correlations are specified via **n -point correlation function**:

$$\langle \delta_1 \delta_2 \dots \delta_n \rangle = \int \delta_1 \delta_2 \dots \delta_n P_n(\delta_1, \delta_2, \dots, \delta_n) d\delta_1 d\delta_2 \dots d\delta_n$$

In particular, we will often use the **two-point correlation function**

$$\xi(x) = \langle \delta_1 \delta_2 \rangle \quad \text{with } x = |\vec{x}_1 - \vec{x}_2|$$

Power Spectrum

It is useful to write $\delta(\vec{x})$ as a Fourier series:

$$\delta(\vec{x}) = \sum_{\vec{k}} \delta_{\vec{k}} e^{i\vec{k} \cdot \vec{x}} \quad \delta_{\vec{k}} = \frac{1}{V} \int \delta(\vec{x}) e^{-i\vec{k} \cdot \vec{x}} d^3 \vec{x}$$

Note that $\delta_{\vec{k}}$ are complex quantities: $\delta_{\vec{k}} = |\delta_{\vec{k}}| e^{i\theta_{\vec{k}}}$

Decomposition in Fourier modes is preserved during **linear evolution**, so that

$$P_n \left(\delta_{\vec{k}_1}, \delta_{\vec{k}_2}, \dots, \delta_{\vec{k}_n} \right) = \prod_{i=1}^n P(\delta_{\vec{k}_i})$$

Thus, statistical properties of $\delta(\vec{x})$ completely specified by $P(\delta_{\vec{k}})$

A **Gaussian random field** is completely specified by first two moments:

$$\begin{aligned} \langle \delta_{\vec{k}} \rangle &= 0 \\ \langle |\delta_{\vec{k}}|^2 \rangle &= P(k) && \text{Power Spectrum} \\ \langle \delta_{\vec{k}} \delta_{\vec{p}} \rangle &= 0 && (\text{for } k \neq p) \end{aligned}$$

The **power spectrum** is Fourier Transform of two-point correlation function:

$$\xi(r) = \frac{1}{(2\pi)^3} \int P(k) e^{i\vec{k} \cdot \vec{r}} d^3 \vec{k} = \frac{1}{2\pi^2} \int_0^\infty P(k) \frac{\sin kr}{kr} k^2 dk$$

Mass Variance

Let $\delta_M(\vec{x})$ be the density field $\delta(\vec{x})$ smoothed (convolved) with a filter of size $R_f \propto [M/\bar{\rho}]^{1/3}$.

Since convolution is multiplication in Fourier space, we have that

$$\delta_M(\vec{x}) = \sum_{\vec{k}} \delta_{\vec{k}} \widehat{W}_M(\vec{k}) e^{i\vec{k} \cdot \vec{x}}$$

with $\widehat{W}_M(\vec{k})$ the FT of the filter function $W_M(\vec{x})$.

The **mass variance** is simply

$$\sigma^2(M) = \langle \delta_M^2 \rangle = \frac{1}{2\pi^2} \int P(k) \widehat{W}_M^2(k) k^2 dk$$

Note that $\sigma^2(M) \rightarrow 0$ if $M \rightarrow \infty$.

Press-Schechter Formalism

In **CDM** universes, density perturbations grow, turn around from Hubble expansion, collapse, and virialize to form **dark matter halo**.

According to **spherical collapse model** the collapse occurs when

$$\delta_{\text{lin}} = \delta_{\text{sc}} \simeq \frac{3}{20} (12\pi)^{2/3} \simeq 1.686$$

δ_{lin} is **linearly extrapolated** density perturbation field

δ_{sc} is **critical overdensity** for **spherical collapse**.

Press-Schechter ansatz: if $\delta_{\text{lin},M}(\vec{x}) > \delta_{\text{sc}}$ then \vec{x} is located in a halo with mass $> M$.

The **probability** that \vec{x} is in a halo of mass $> M$ therefore is

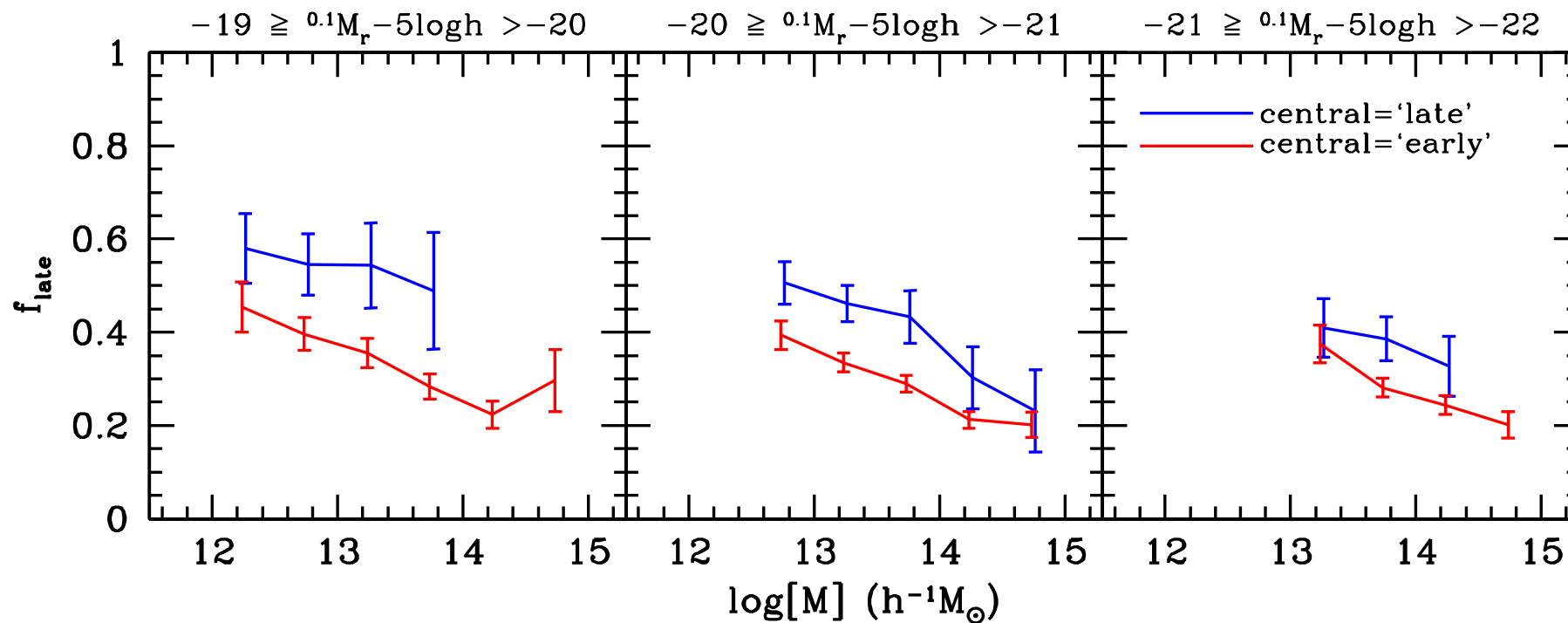
$$P(\delta_{\text{lin},M} > \delta_{\text{sc}}) = \frac{1}{\sqrt{2\pi}\sigma(M)} \int_{\delta_{\text{sc}}}^{\infty} \exp\left(-\frac{\delta^2}{2\sigma^2(M)}\right) d\delta$$

The **Halo Mass Function**, then becomes

$$n(M)dM = \frac{\bar{\rho}}{M} \frac{dP}{dM} dM = \sqrt{\frac{2}{\pi}} \frac{\bar{\rho}}{M^2} \left| \frac{d \ln \sigma}{d \ln M} \right| \sqrt{\nu} e^{-\nu/2}$$

where $\nu = \delta_{\text{sc}}^2 / \sigma^2(M)$, and a ‘fudge-factor’ 2 has been added.

Galactic Conformity



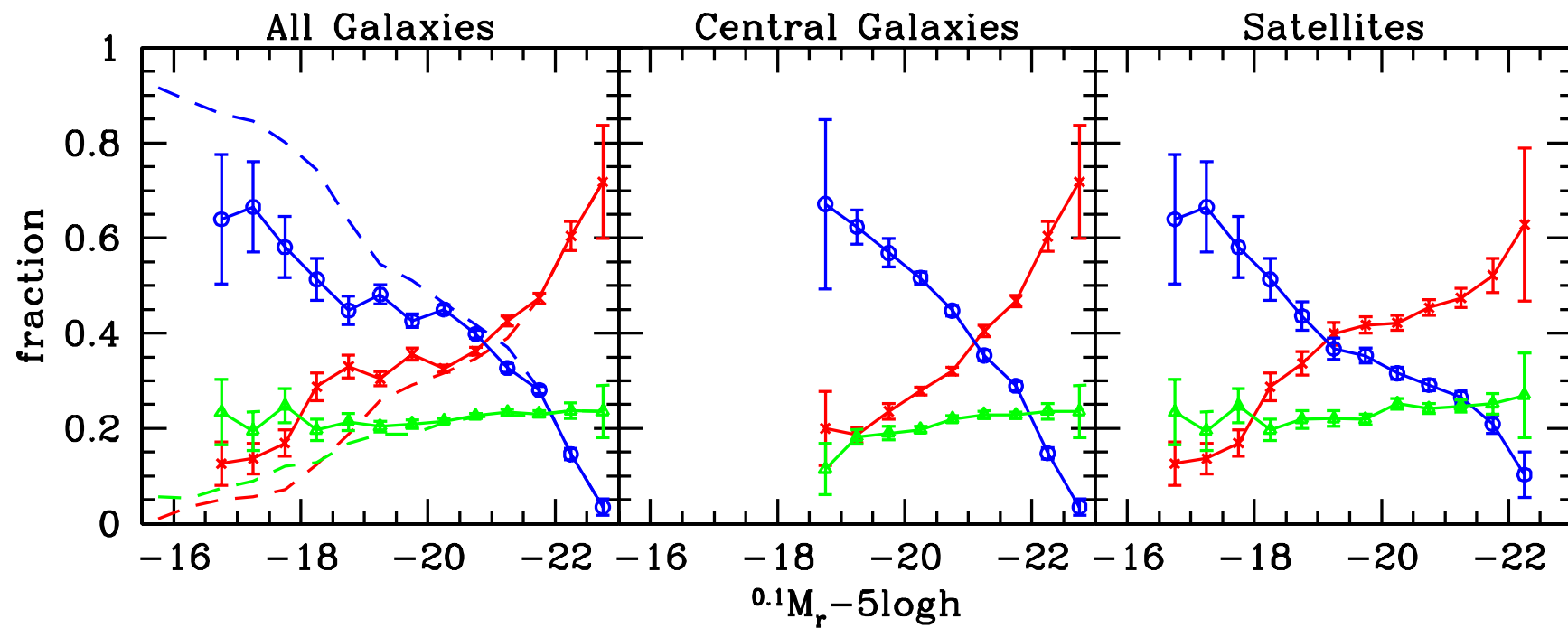
Late type 'centrals' have preferentially **late type** satellites, and vice versa.

Satellite galaxies '**adjust**' themselves to properties of their central galaxy

Galactic Conformity present over large ranges in luminosity and halo mass.

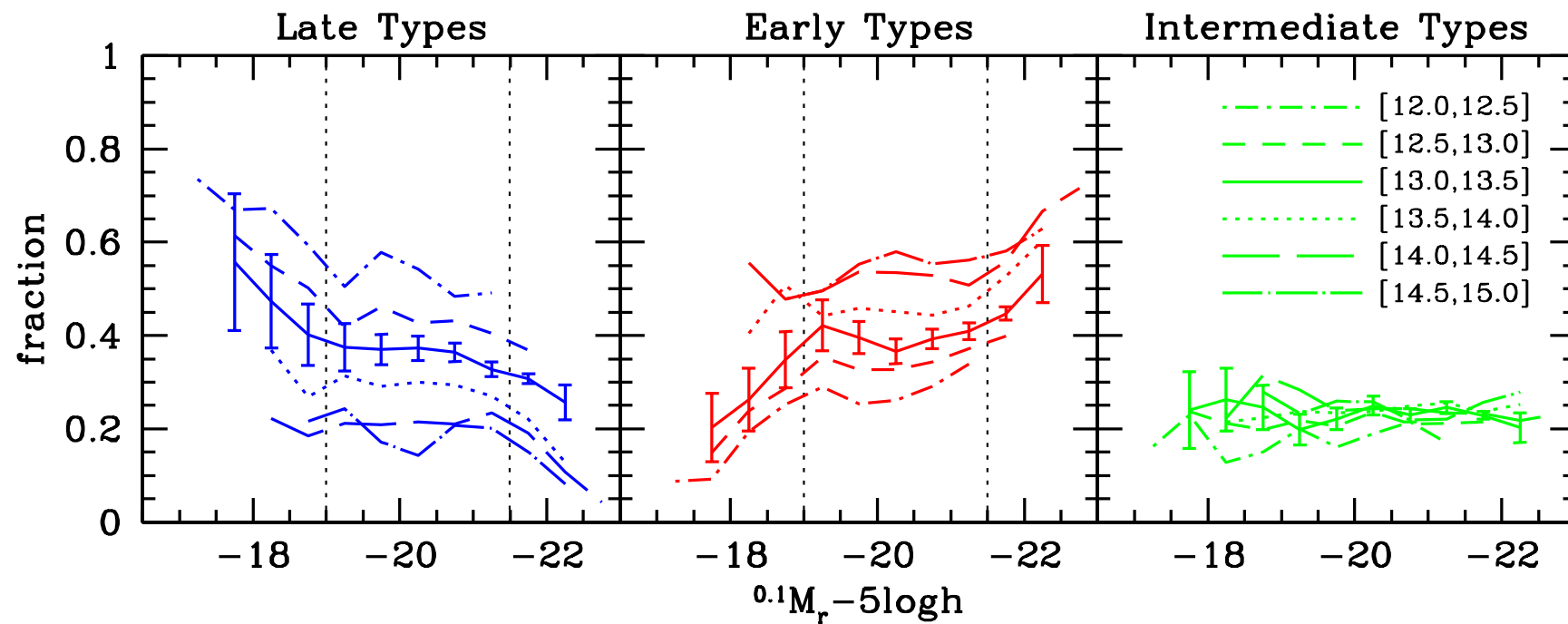
(Weinmann, vdB, Yang & Mo, 2006)

Luminosity Dependence



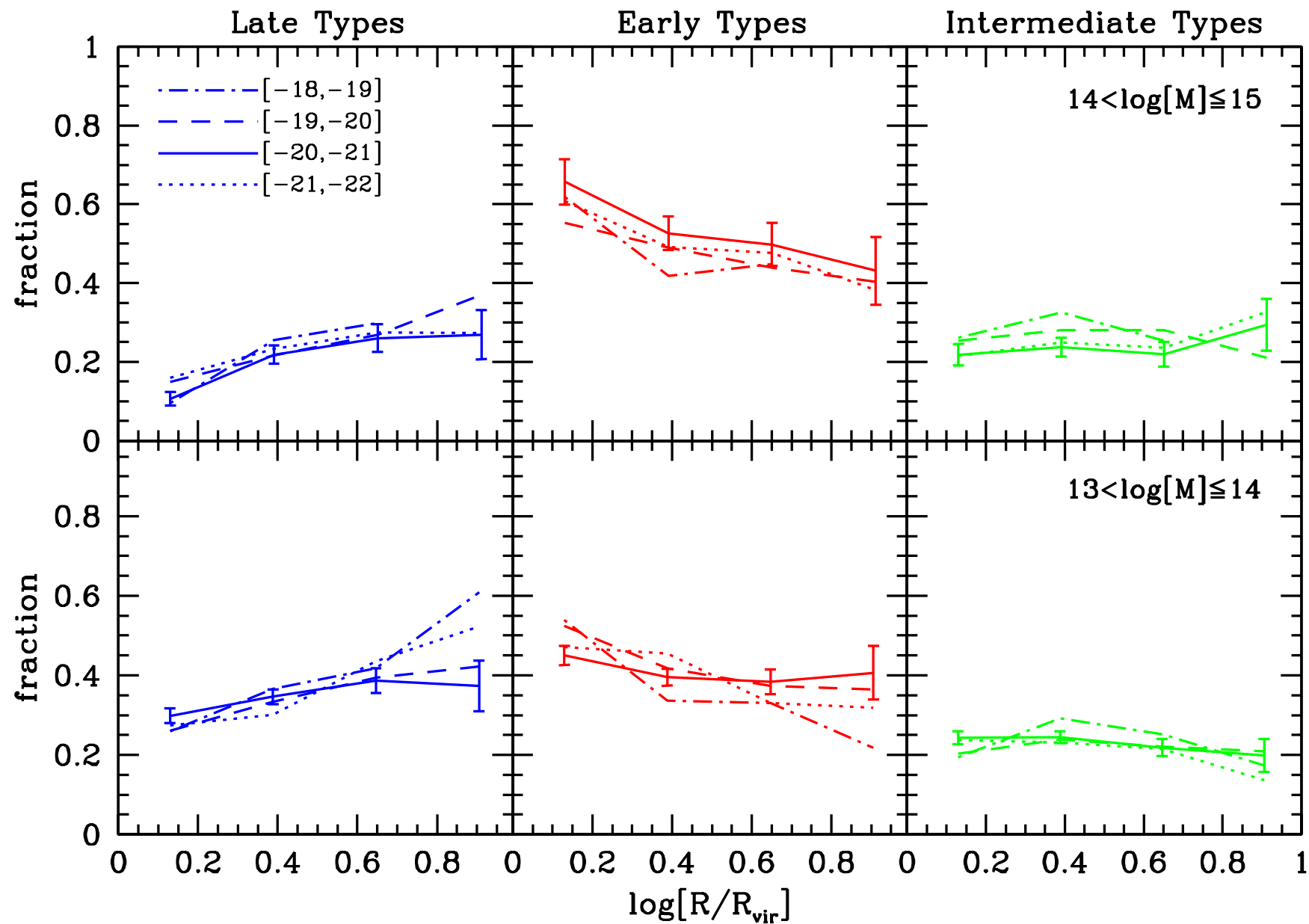
(Weinmann, vdB, Yang & Mo, 2005)

Mass-Luminosity Dependence



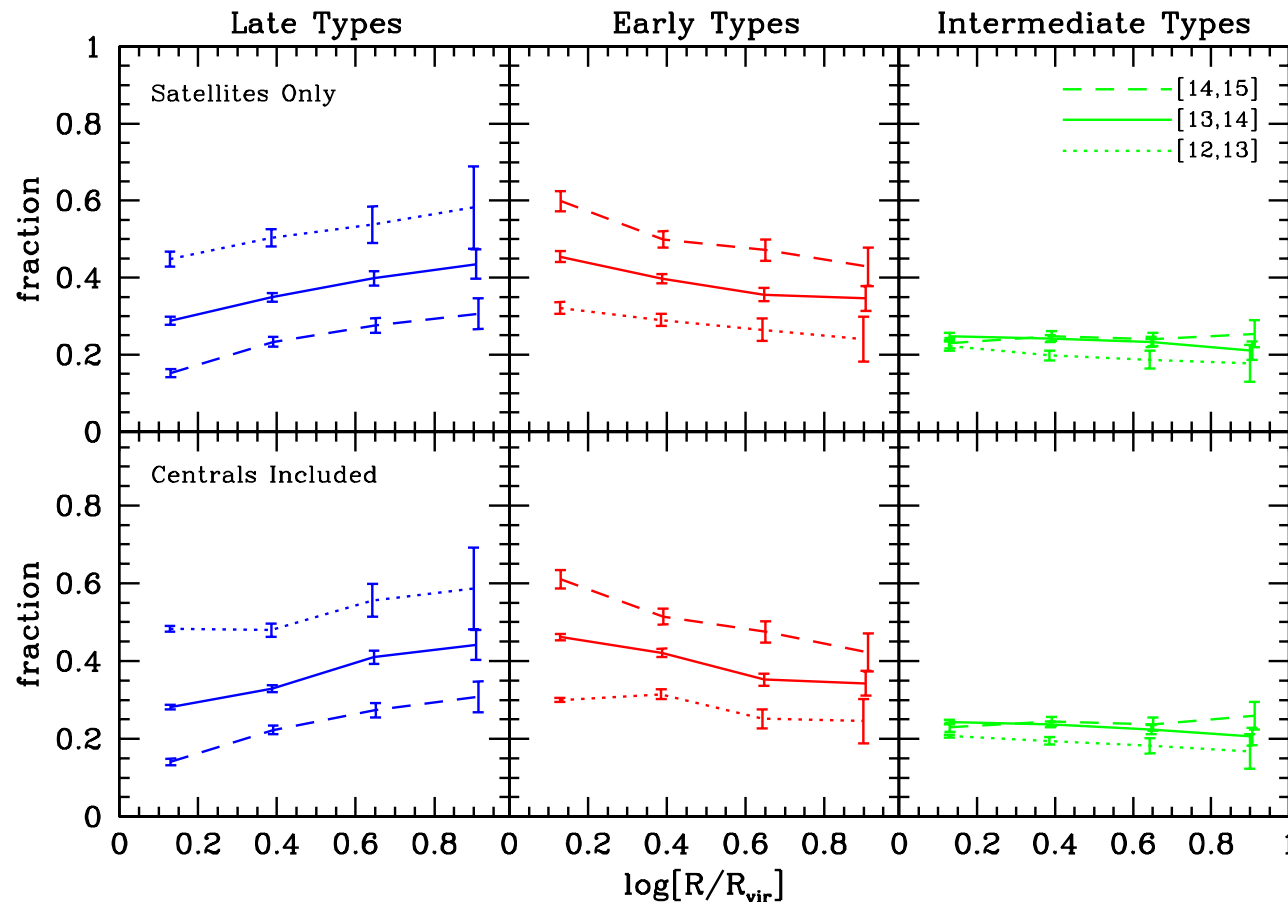
(Weinmann, vdB, Yang & Mo, 2005)

Radial Dependence



(Weinmann, vdB, Yang & Mo, 2005)

Dependence on Group-centric Radius



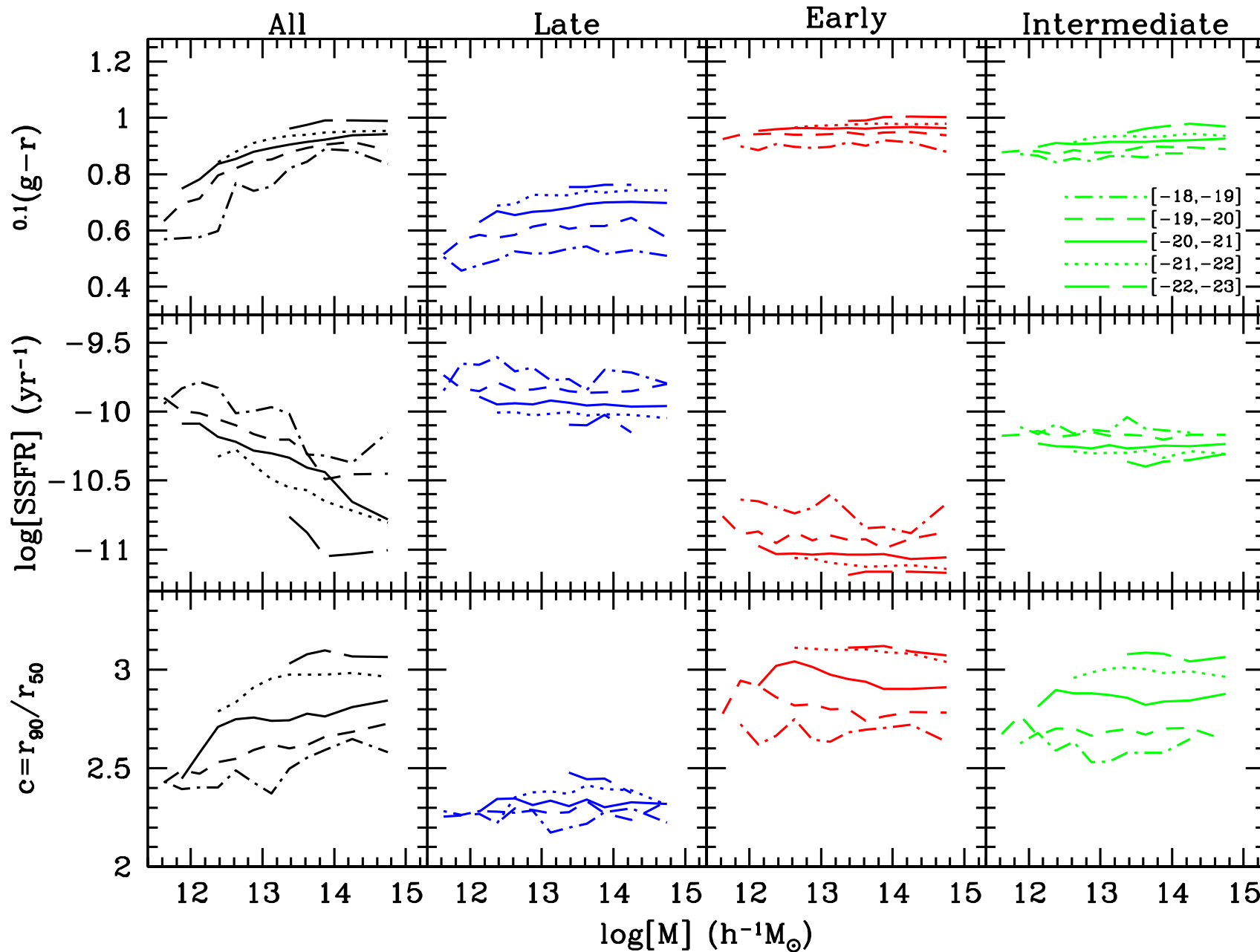
As noticed before, the late type fraction **of satellites** increases with radius.
This trend is **independent of halo mass**!

Inconsistent with previous studies, but these **included** central galaxies.

Our results rule out group- and cluster-specific processes such as
ram-pressure stripping and **harassment**: **nature** rather than **nurture**!

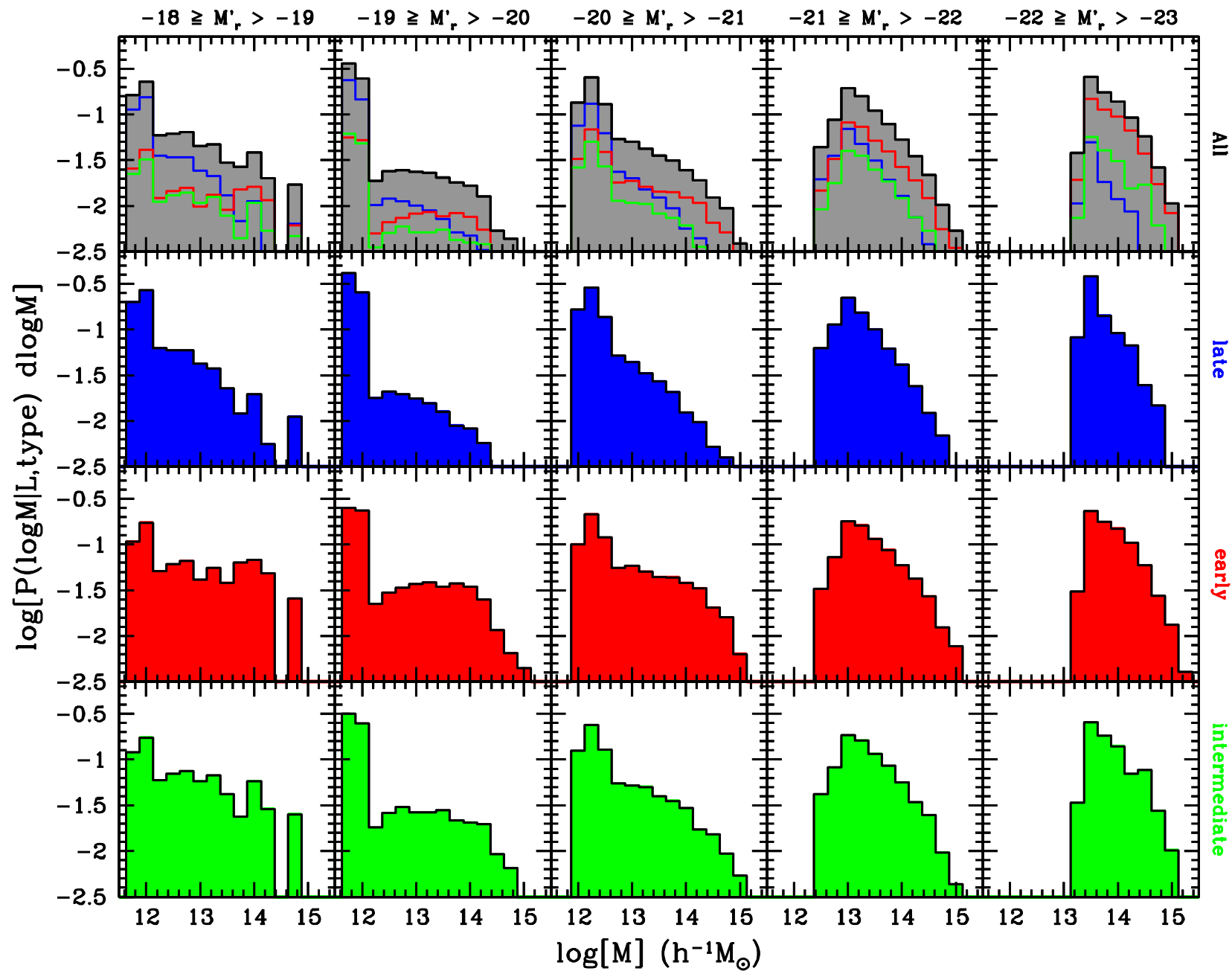
(Weinmann, vdB, Yang & Mo, 2005)

Median Properties

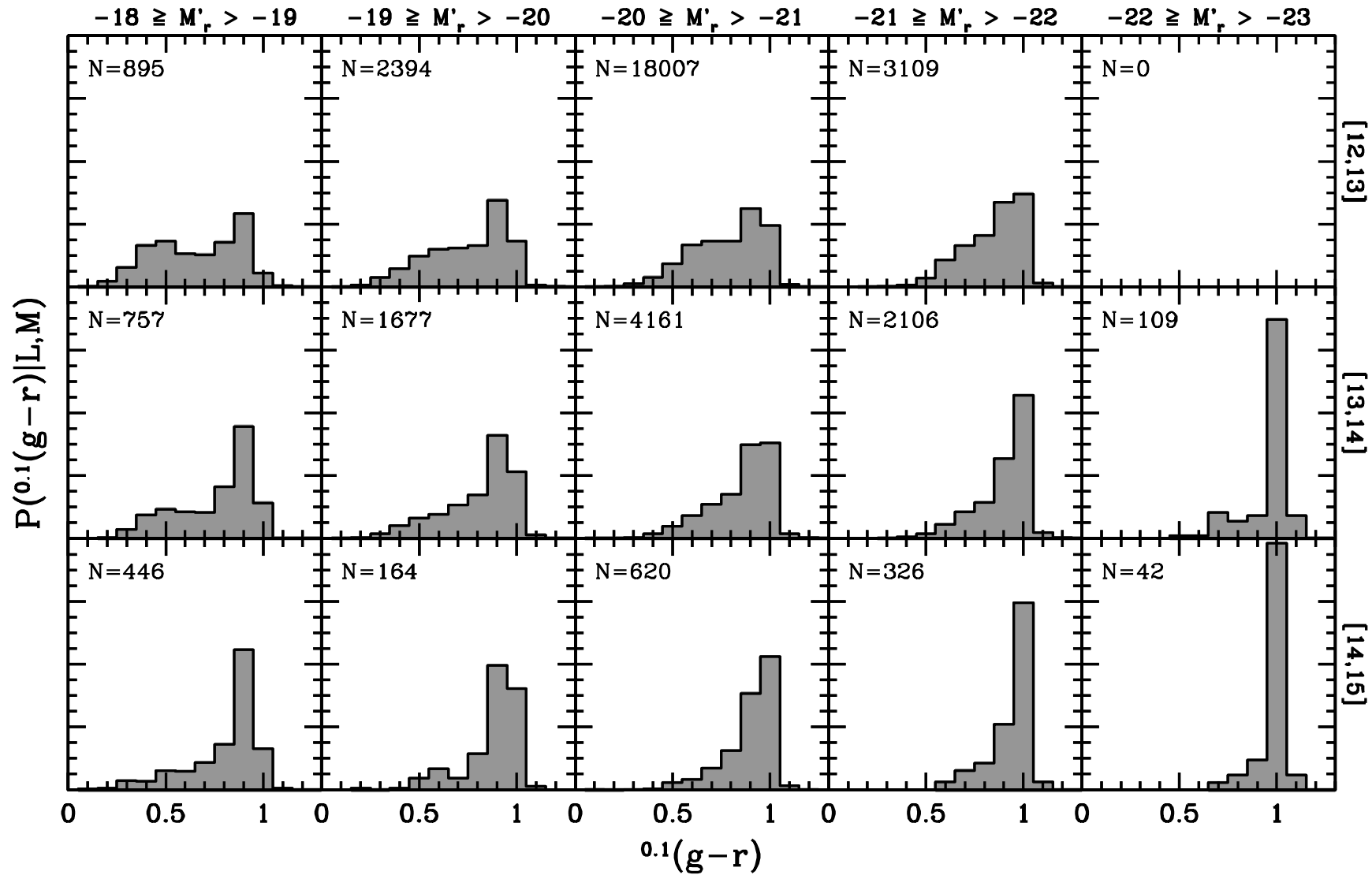


(Weinmann, vdB, Yang & Mo, 2005)

Conditional Mass Function

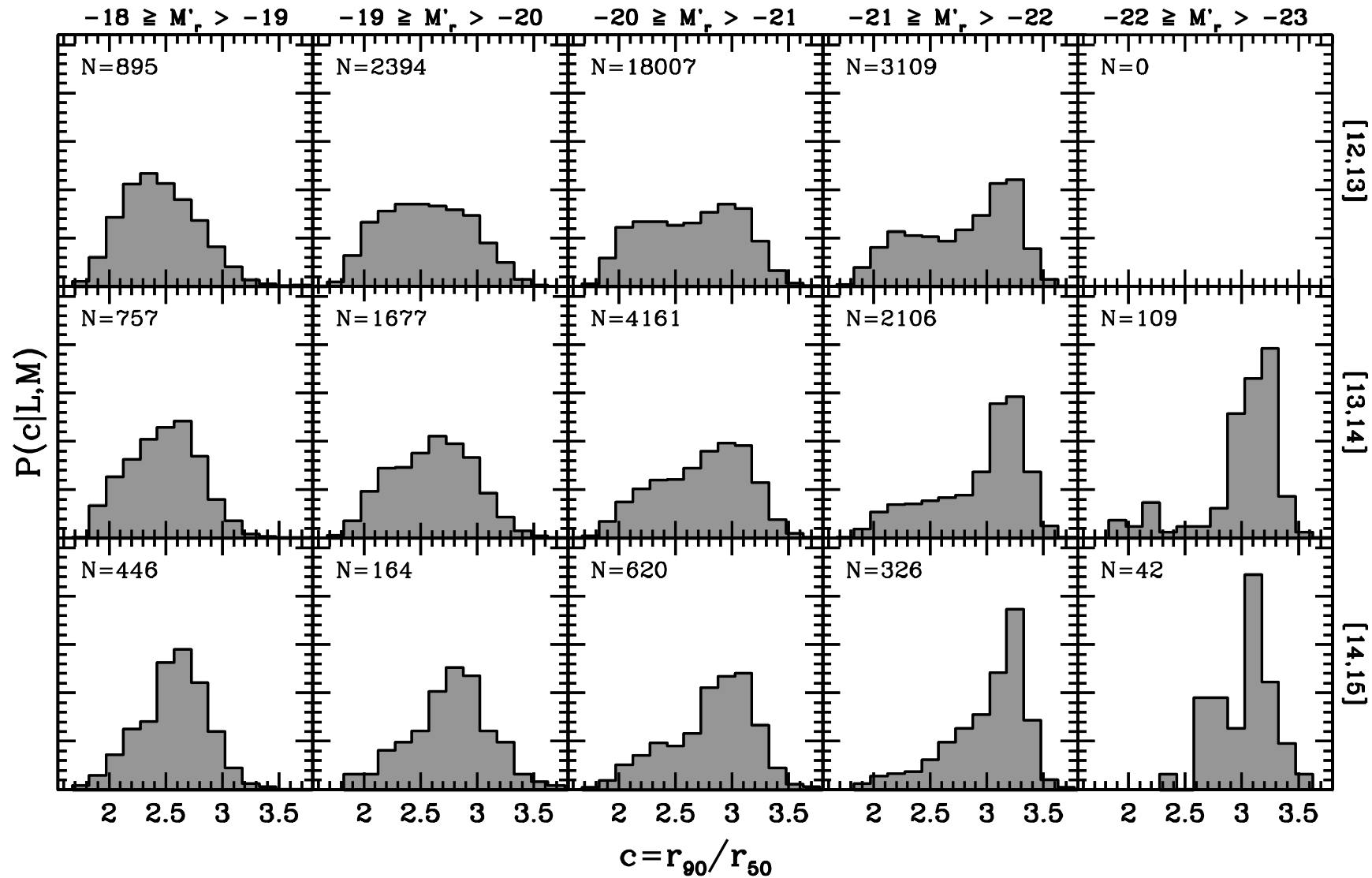


Conditional Colour Function



(Weinmann, vdB, Yang & Mo, 2005)

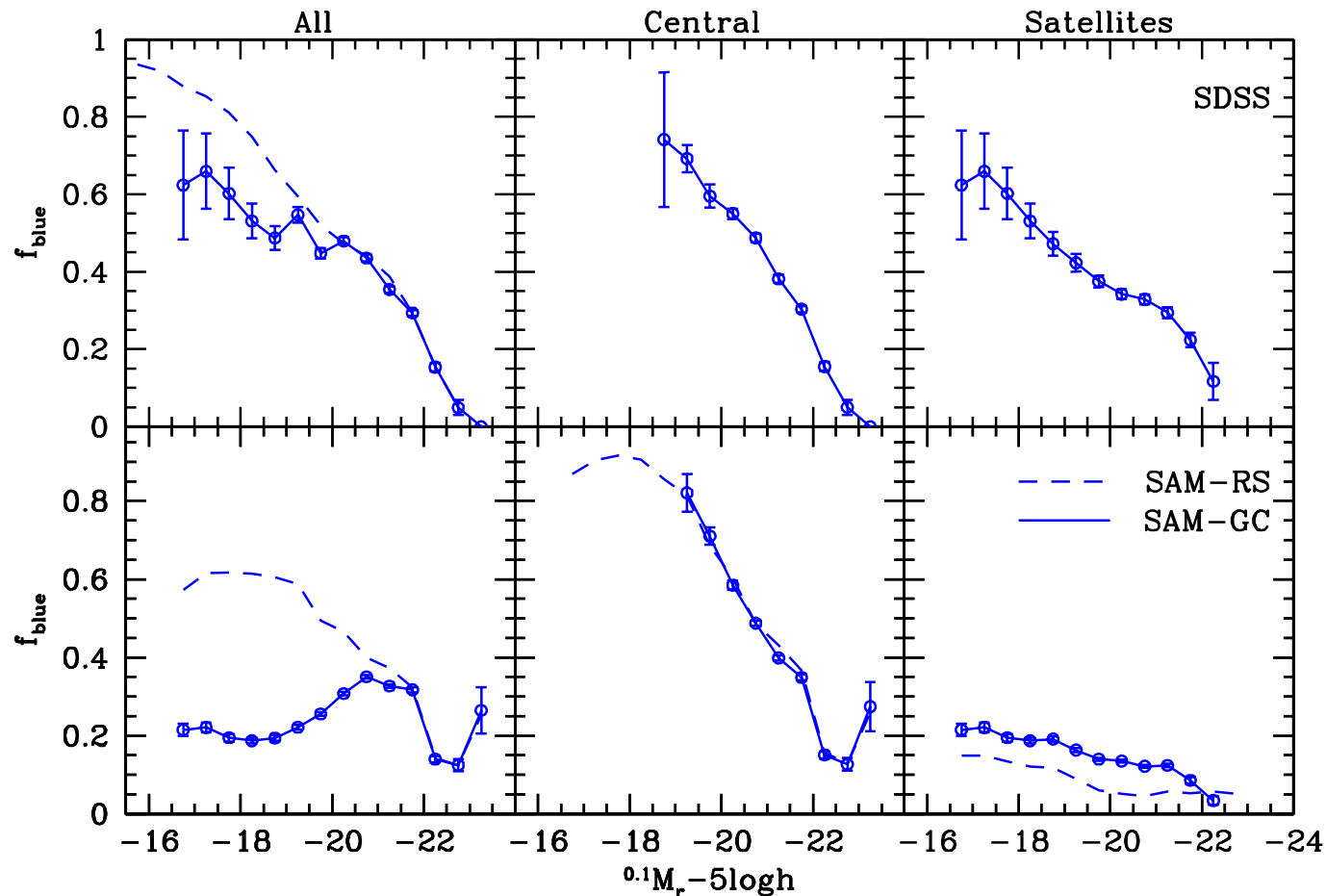
Conditional Concentration Function



(Weinmann, vdB, Yang & Mo, 2005)

Blue Fraction as Function of Luminosity

In **SAM** virtually all satellites are **red**, contrary to **SDSS**, where the fraction of **red** satellites decreases with luminosity.

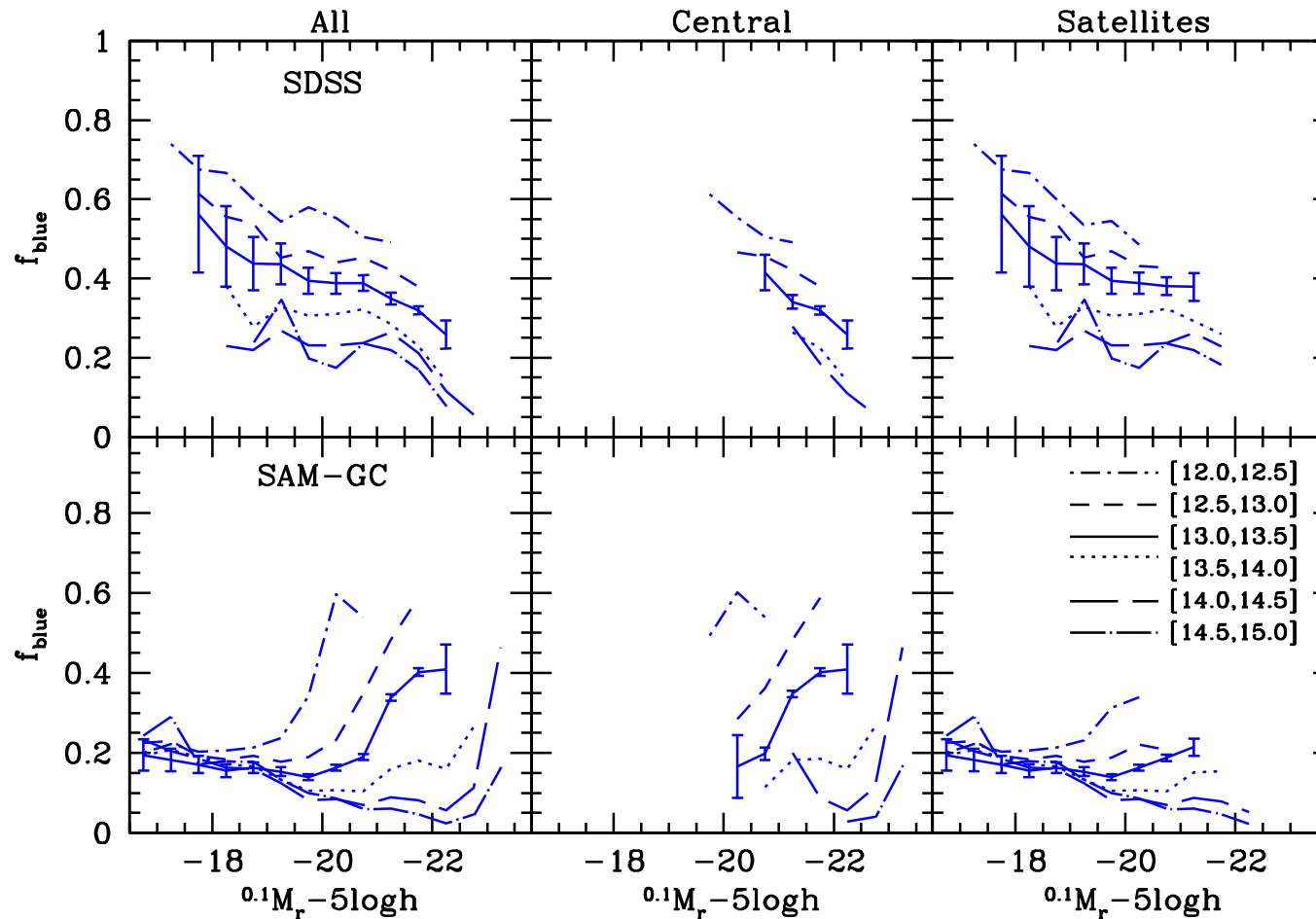


In **SAM**, satellites stripped of hot gas halo immediately after accretion

▷ This **strangulation** is too efficient.

Blue Fraction as Function of Halo Mass

To allow for fair comparison, we run our Group Finder over **SAM**.



Problem with central galaxies: At fixed halo mass, **blue fraction** increases with **L** in **SAM**, but decreases with **L** in **SDSS**.

▷ Modelling of **AGN feedback** is not yet entirely correct

The Importance of Satellite Galaxies

Kinematics

- Satellites sample large radii \Rightarrow virial mass estimator
- Only few satellites per halo \Rightarrow Need to stack many host/satellite pairs
- Beware of Interlopers & Observational Biases

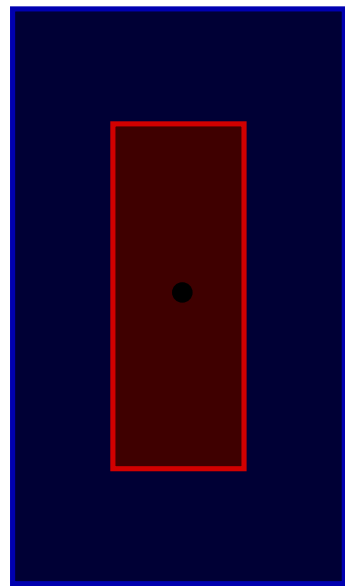
Abundances

- Halo Occupation Numbers \Leftrightarrow Conditional Luminosity Function
(Yang, Mo & van den Bosch 2003; van den Bosch, Yang & Mo 2003)
- Comparison with dark matter subhaloes
(Moore et al. 1999; Klypin et al. 1999; Vale & Ostriker 2004; Kravtsov et al. 2004)

Radial Distribution

- Dark Matter Subhaloes show spatial anti-bias
(Klypin et al. 1999; Ghigna et al. 2000; De Lucia et al. 2003; Diemand et al. 2004)
- Impact of tidal stripping, dynamical friction, harassment etc.
(Moore et al. 1998; Mayer et al. 2001; Taffoni et al. 2003)

Selecting hosts & satellites



redshift

HOSTS *At least f_h times brighter than any other galaxy in blue volume.*

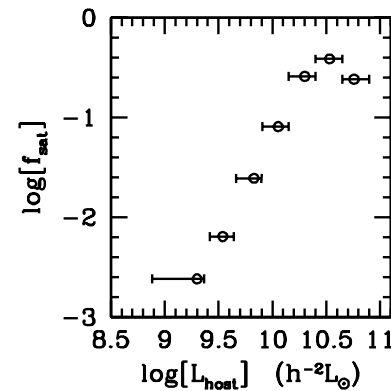
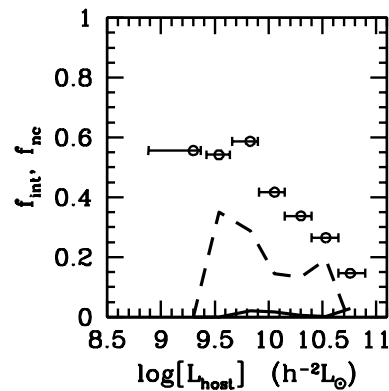
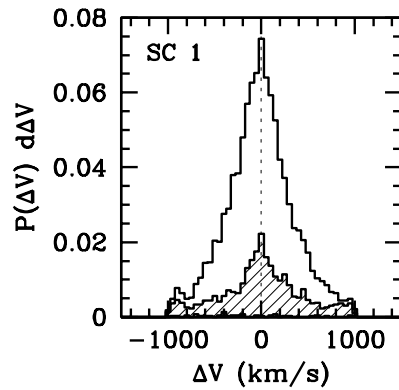
SATS *In red volume and at least f_s times fainter than host.*

Projected
Distance

All previous studies used very **conservative** selection criteria: $f_h = 2$, $f_s = 4$, $\Delta V_h = \Delta V_s = 1000 \text{ km s}^{-1}$, $R_h = 2h^{-1} \text{ Mpc}$, and $R_s = 0.5h^{-1} \text{ Mpc}$. (McKay et al. 2002; Brainerd & Specian 2003; Prada et al. 2003)

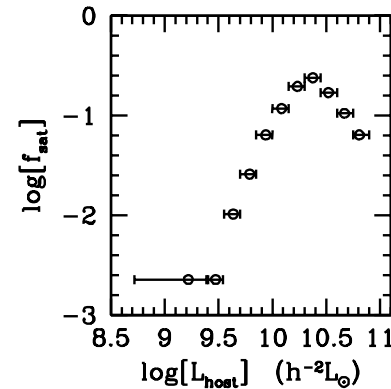
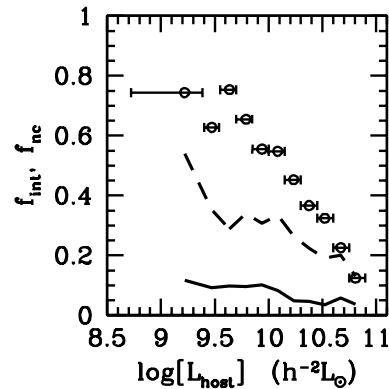
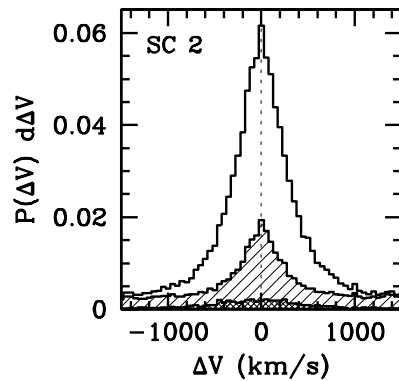
Use **Mock Galaxy Redshift Surveys**, constructed from **CLF** to test selection criteria.

Selecting hosts & satellites



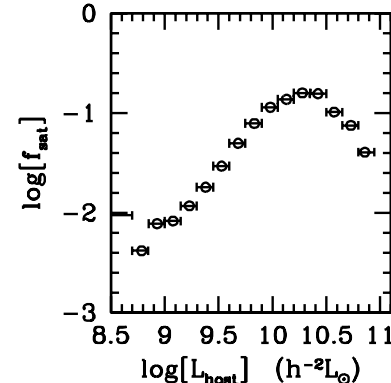
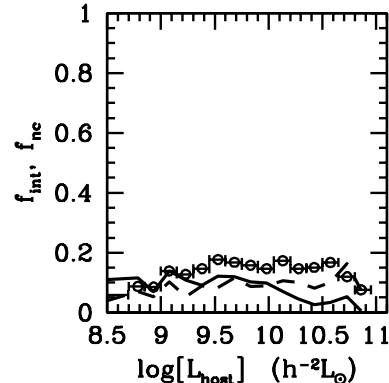
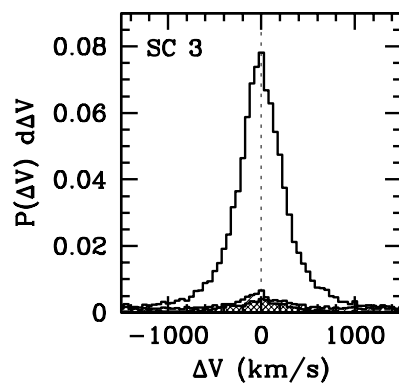
$f_h=2.0$
 $f_s=4.0$
 $R_h=2.0$ chimp
 $R_s=0.5$ chimp
 $dV_h=1000$ km/s
 $dV_s=1000$ km/s

$N_h=1851$
 $N_s=3876$



$f_h=1.0$
 $f_s=1.0$
 $R_h=2.0$ chimp
 $R_s=0.5$ chimp
 $dV_h=2000$ km/s
 $dV_s=2000$ km/s

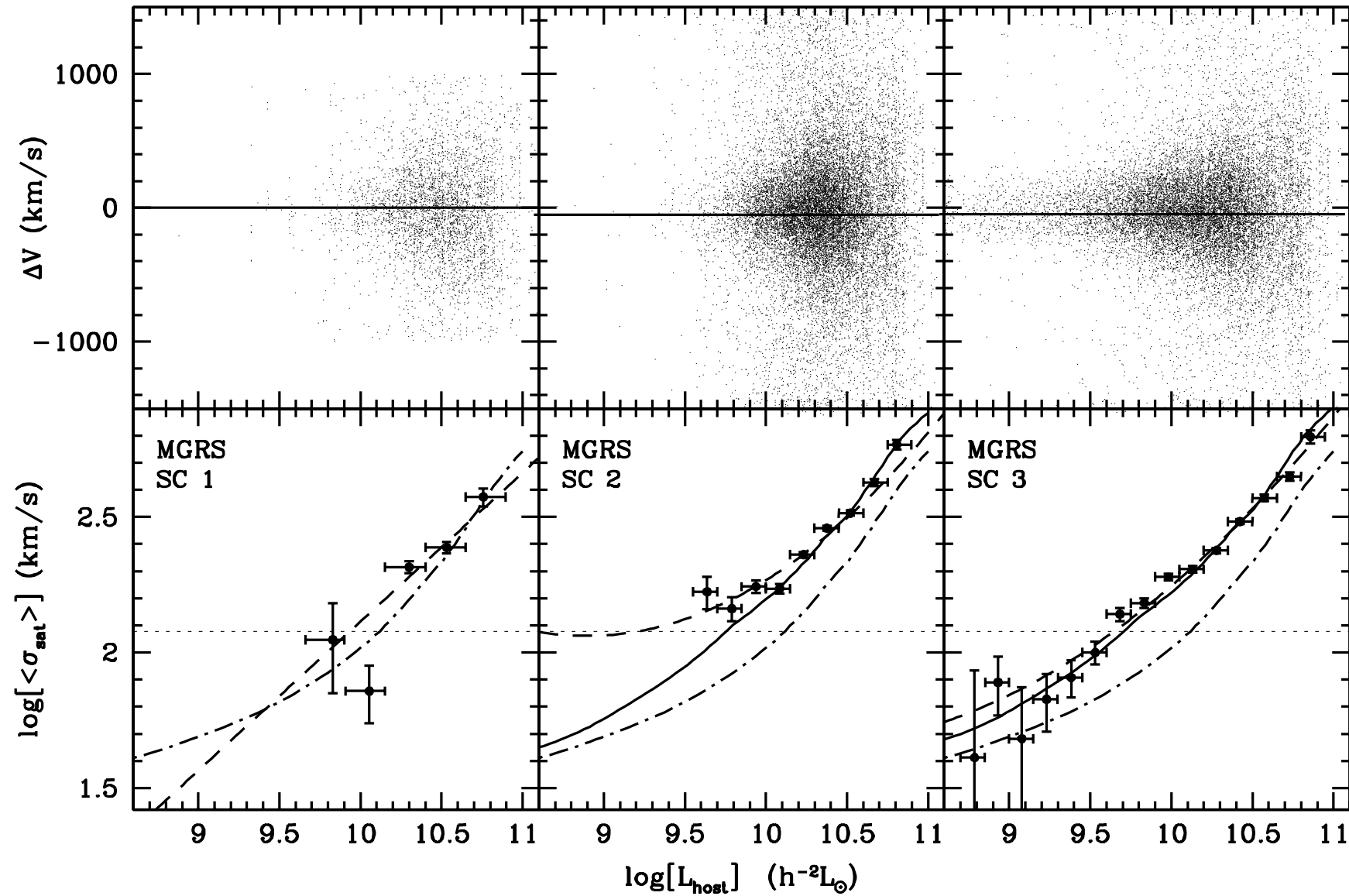
$N_h=7863$
 $N_s=19099$



$f_h=1.0$
 $f_s=1.0$
 assume $\sigma(L)$
 iterate

$N_h=10483$
 $N_s=16750$

Measuring Satellite Kinematics



Interpreting Satellite Kinematics

Velocity dispersion of satellite galaxies follows from **Jeans Equation**:

$$\sigma_{\text{sat}}^2(r) = \frac{1}{n_{\text{sat}}(r)} \int_r^\infty n_{\text{sat}}(r') \frac{\partial \Phi}{\partial r}(r') dr'$$

The **halo averaged** expectation value:

$$\langle \sigma_{\text{sat}} \rangle_M = \frac{4\pi}{\langle N_{\text{sat}} \rangle_M} \int_0^{r_{\text{vir}}} n_{\text{sat}}(r) \sigma_{\text{sat}}(r) r^2 dr$$

Expectation value for the number of satellites follows from **CLF**:

$$\langle N_{\text{sat}} \rangle_M = \int_{L_1}^\infty \Phi(L|M) dL - 1$$

Scatter in relation between M and host luminosity L_h :

$$\langle \sigma_{\text{sat}} \rangle(L_h) = \int_0^\infty P(M|L_h) \langle \sigma_{\text{sat}} \rangle_M dM$$

Stacking host-satellite pairs yields **satellite-weighted mean**:

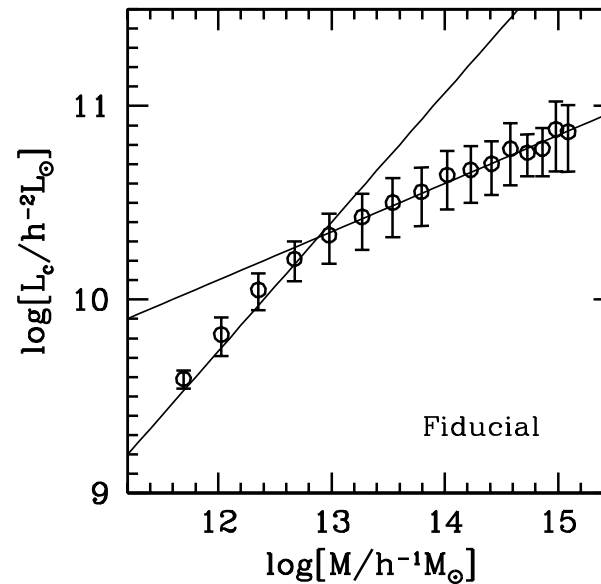
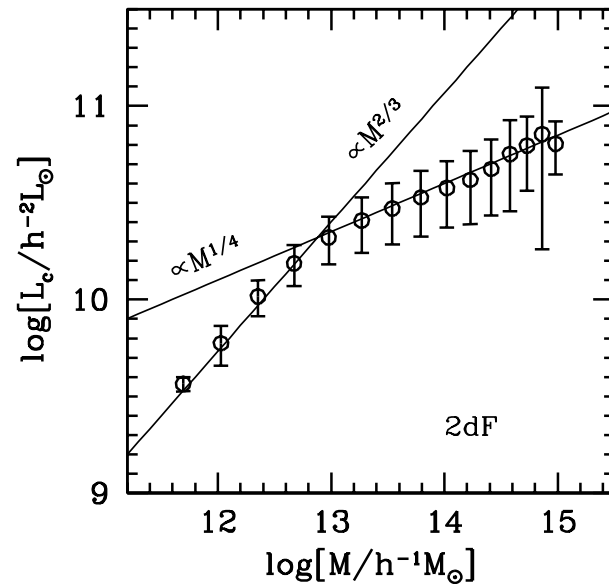
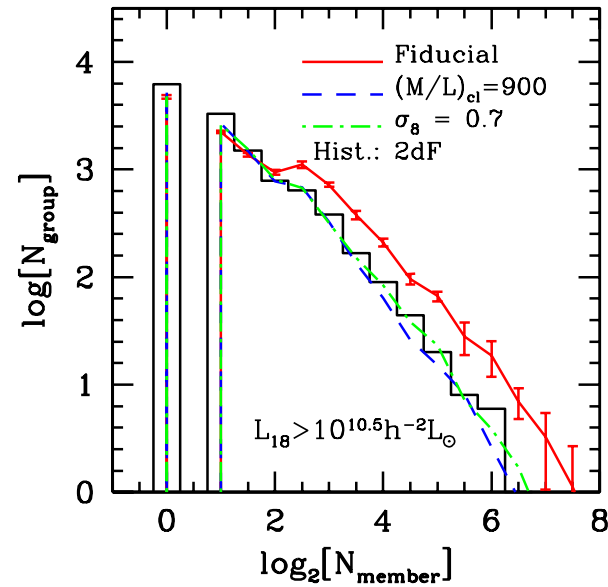
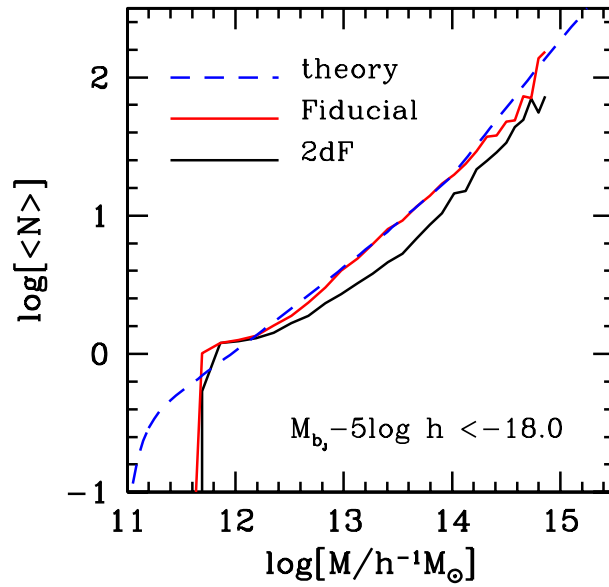
$$\langle \sigma_{\text{sat}} \rangle(L_h) = \frac{\int_0^\infty P(M|L_h) \langle \sigma_{\text{sat}} \rangle_M \langle N_{\text{sat}} \rangle_M dM}{\int_0^\infty P(M|L_h) \langle N_{\text{sat}} \rangle_M dM}$$

Accounting for **flux-limited surveys**:

$$\langle \sigma_{\text{sat}} \rangle(L_h) = \frac{1}{V} \int_0^\Omega d\Omega \int_0^{z_{\text{max}}} dz \frac{dV}{d\Omega dz} \langle \sigma_{\text{sat}}(L_h, z) \rangle$$

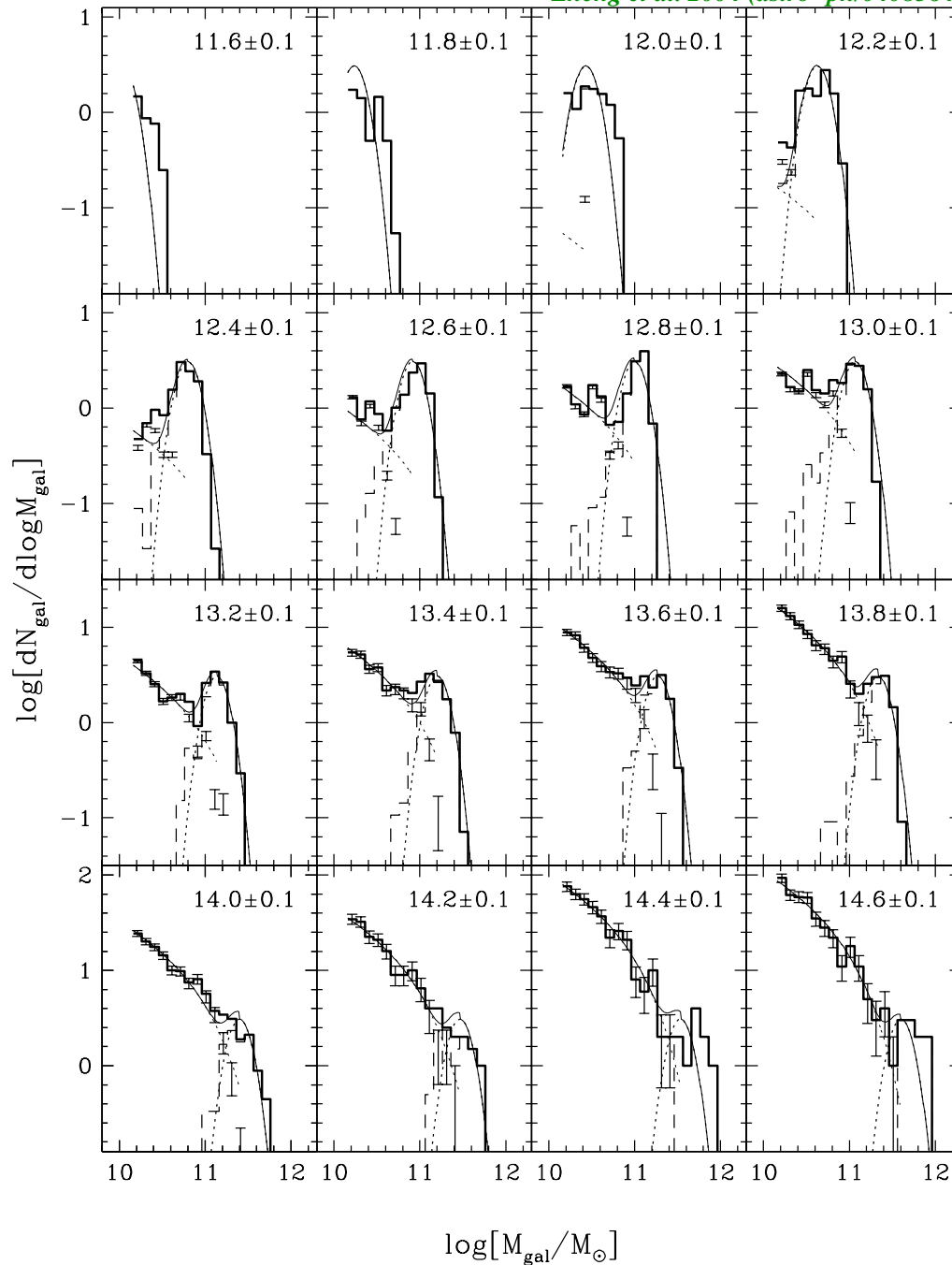
You need CLF to properly interpret satellite kinematics

Various Statistics of Galaxy Groups



The shape of the CLF

Zheng et al. 2004 (astro-ph/0408564)



Using semi-analytical models
Zheng et al (2004) computed
the conditional baryonic
mass function (CMF).

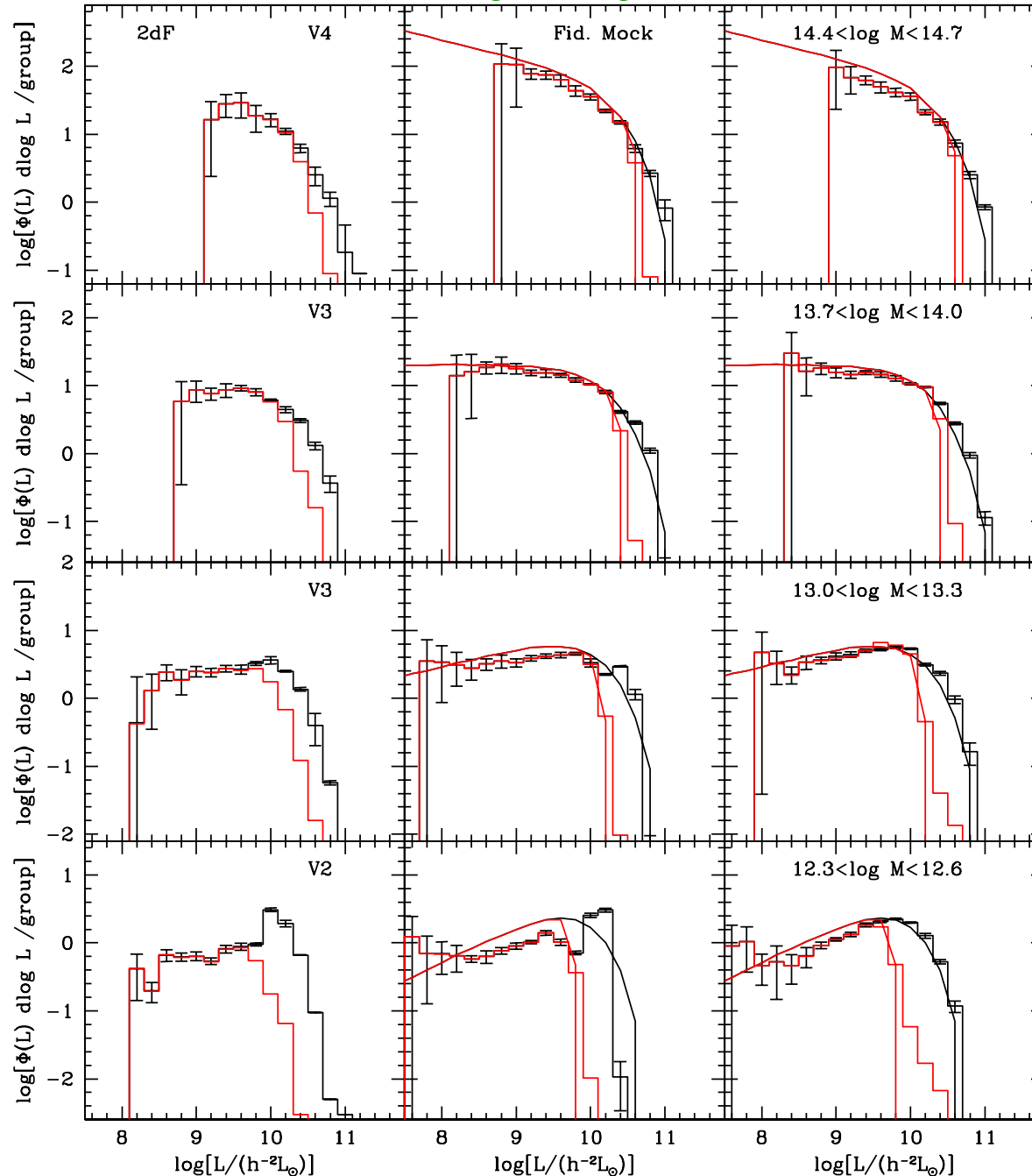
For group-sized haloes
the CMF does not follow
a Schechter function.

Instead, the CMF is better
fit with a Gaussian (for the
central galaxies) plus a
power-law describing the
satellite galaxies.

**Does the true CLF
have a similar shape??**

Direct Determination of CLF from Groups

Yang, Mo, Jing & vdB, 2005, MNRAS, 358, 217



We determined CLF directly from groups in 2dFGRS & mocks.

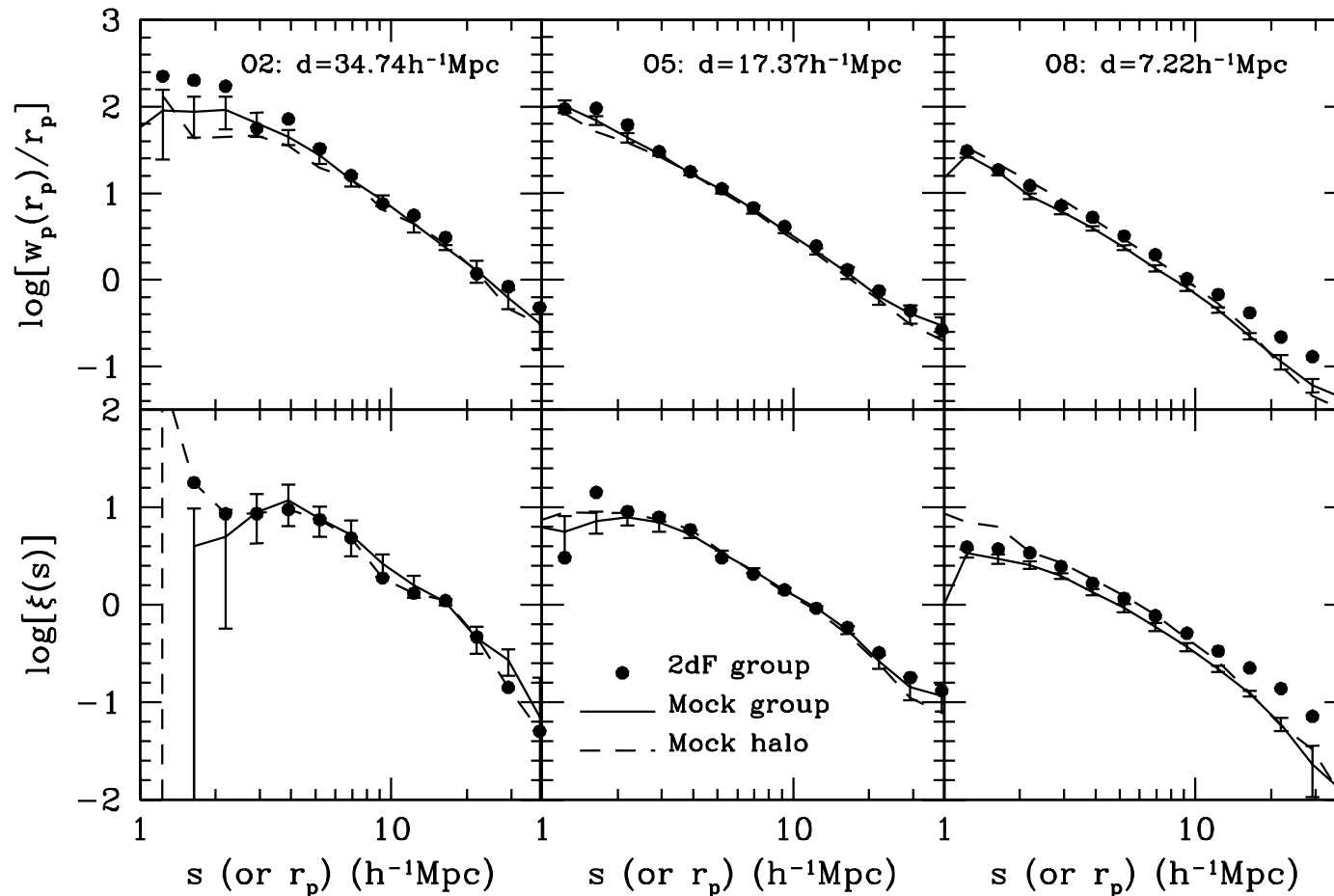
Group-sized haloes have Schechter CLF.

Galaxy-sized haloes reveal Gaussian peak of central galaxies.

Mocks show similar behavior, pointing to an artefact due to mass estimator.

Data is consistent with Schechter CLF!

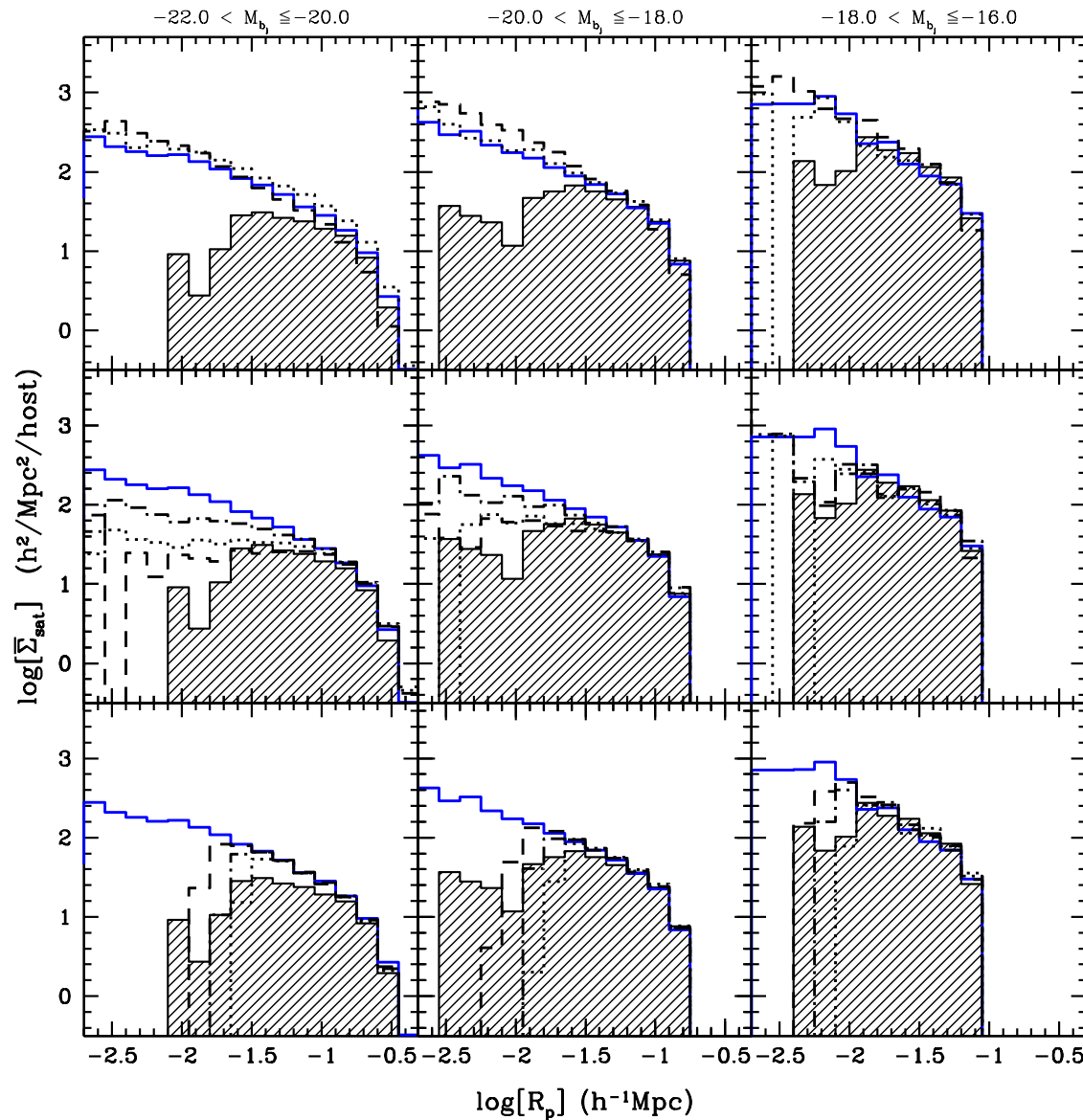
Probing Clustering of Dark Matter Haloes



Yang, Mo, vdB & Jing 2005, MNRAS, 357, 608

- The group-group correlation function **directly** reflects the halo-halo correlation function (**no issues with galaxy bias**)
- Promising tool to constrain **cosmological parameters**

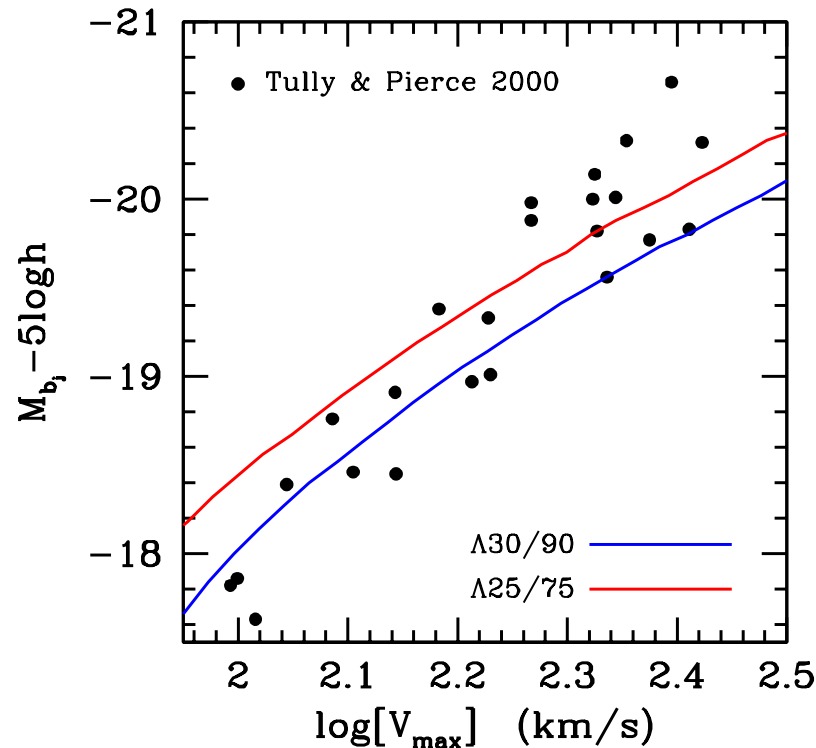
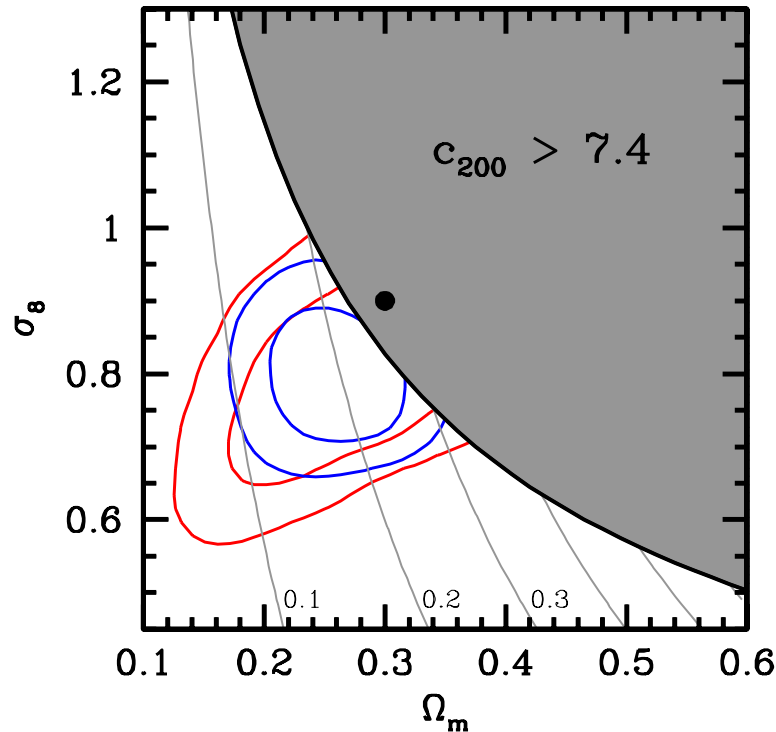
Radial Distribution of Satellite Galaxies



Consistent with **no spatial bias**, but only if $R_{\text{gal}} \simeq 15h^{-1} \text{ kpc} L_{10}^{1/3}$

Concordance on Galactic Scales?

$$\Omega_m = 0.25 \pm 0.04 \quad \sigma_8 = 0.78 \pm 0.06$$



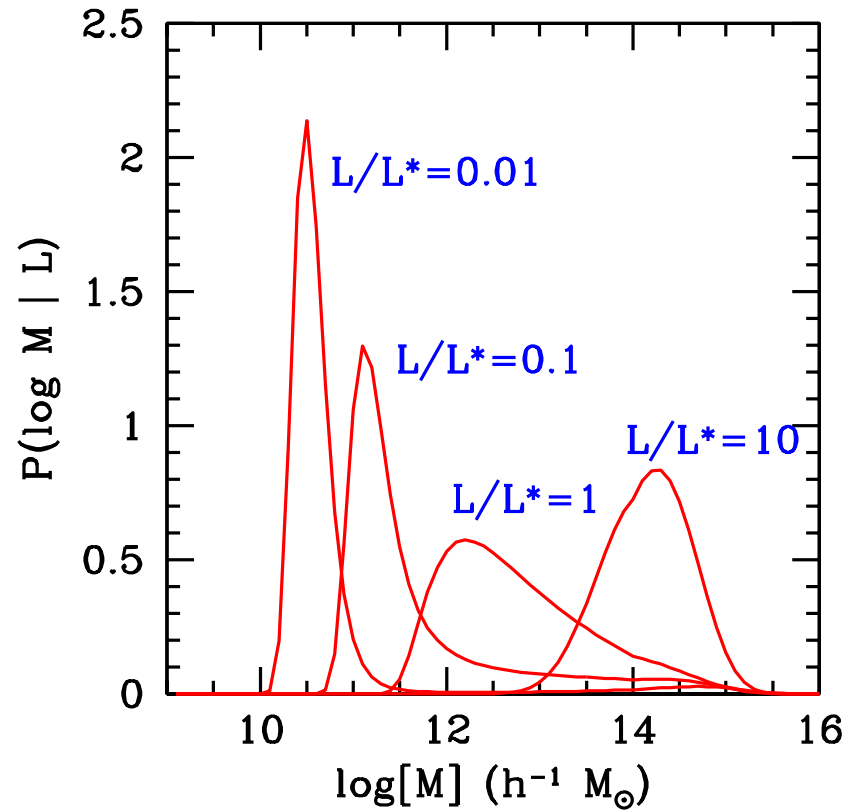
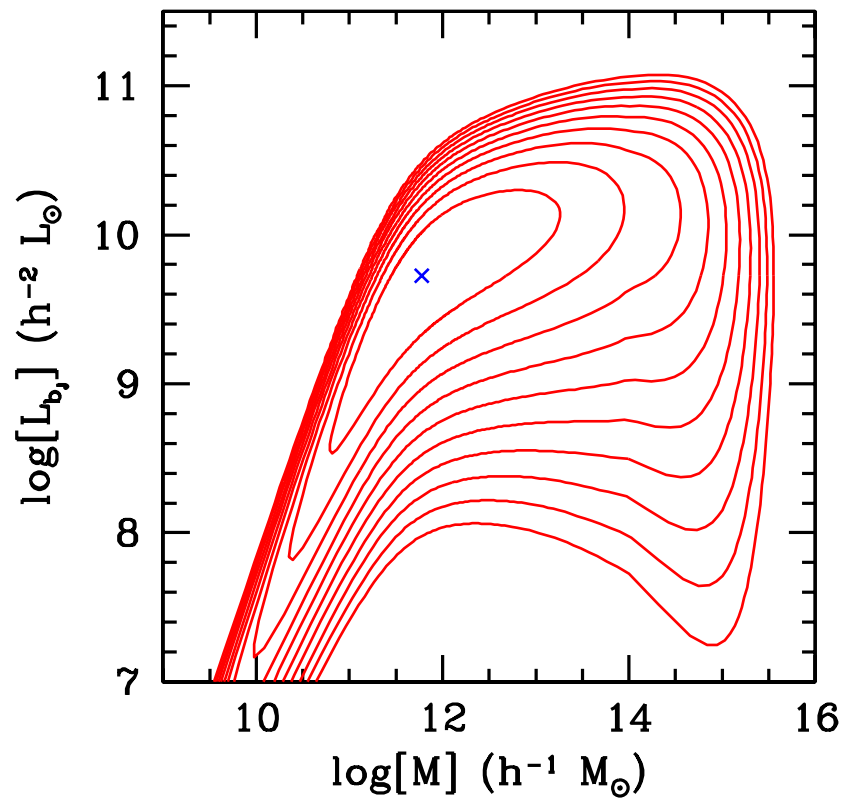
Cosmologies with lower Ω_m and lower σ_8 yield dark matter haloes that are significantly less concentrated. This

- Alleviates problem with **rotation curves** of dwarf and LSB galaxies.
- Results in a **TF zero-point** that is $\sim 0.3 - 0.5$ magnitudes brighter.

The Galaxy Phone Book

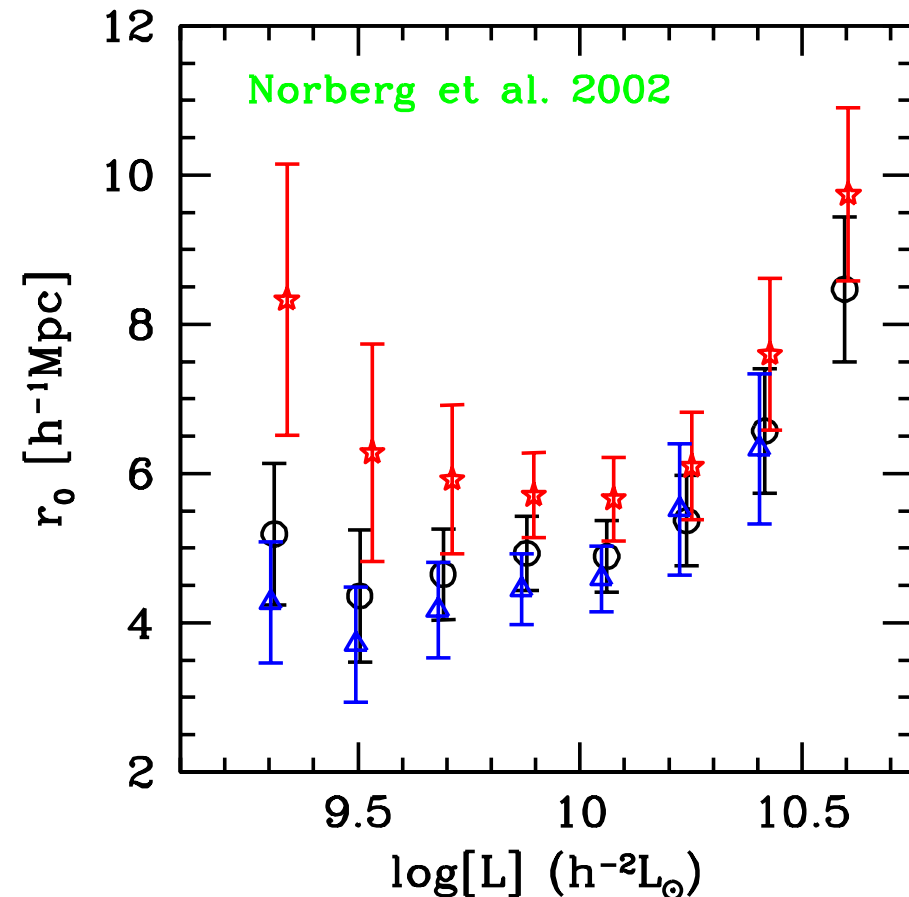
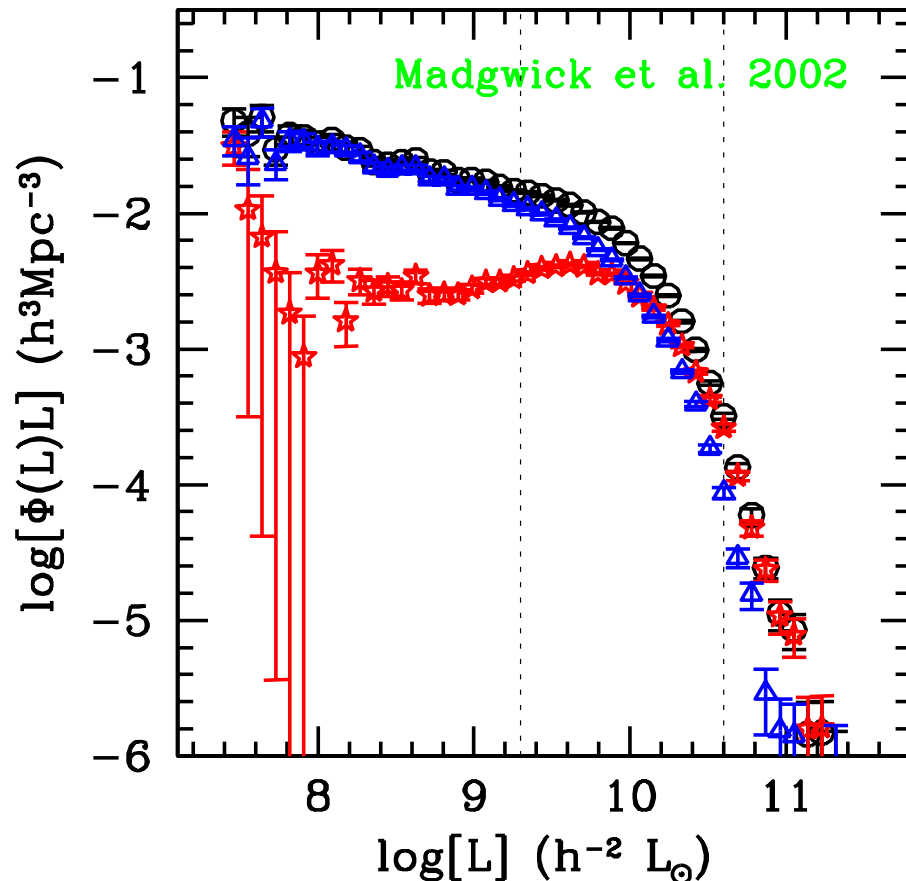
$$P(L, M) dL dM = \frac{1}{\bar{\rho}_L} n(M) \Phi(L|M) L dL dM$$

$$P(M|L)dM = \frac{\Phi(L|M) n(M) dM}{\Phi(L)}$$



50% of light is produced in haloes $M \lesssim 2 \times 10^{12} h^{-1} \text{ M}_\odot$.

Luminosity & Correlation Functions

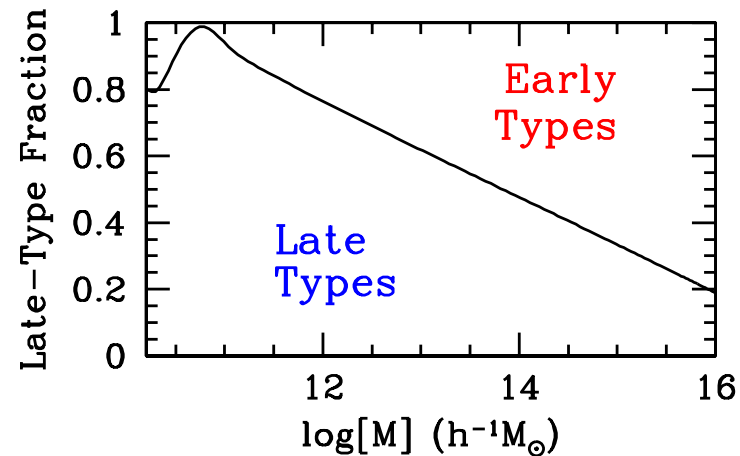
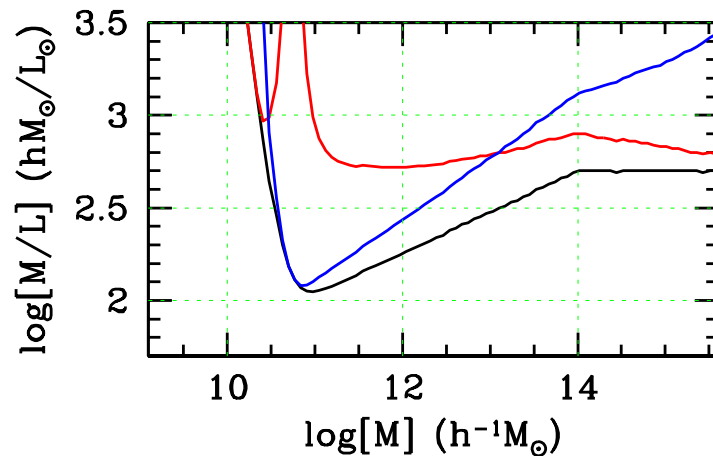
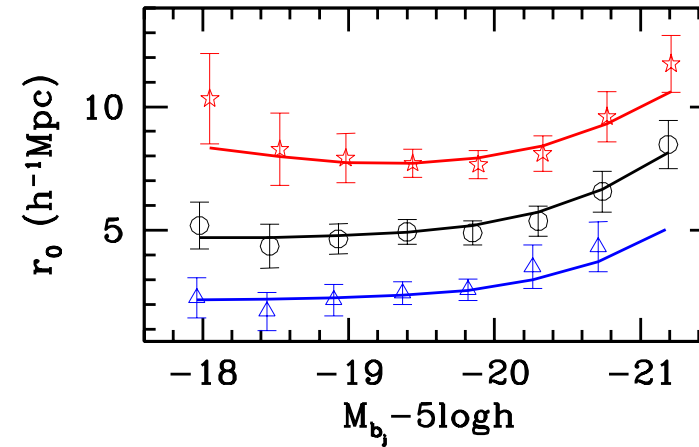
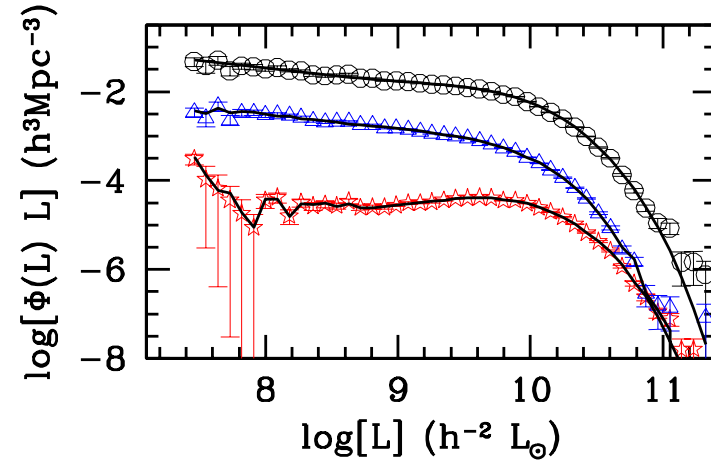


- On average, **early-type** galaxies are more luminous and more strongly clustered than **late-type** galaxies.
- In general, more luminous galaxies are more strongly clustered.

REMINDER: Correlation length r_0 defined by $\xi(r_0) = 1$

The Concordance Cosmology

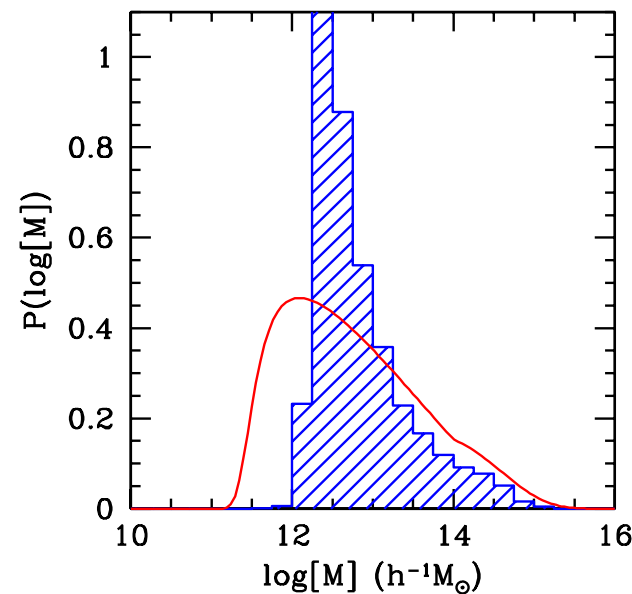
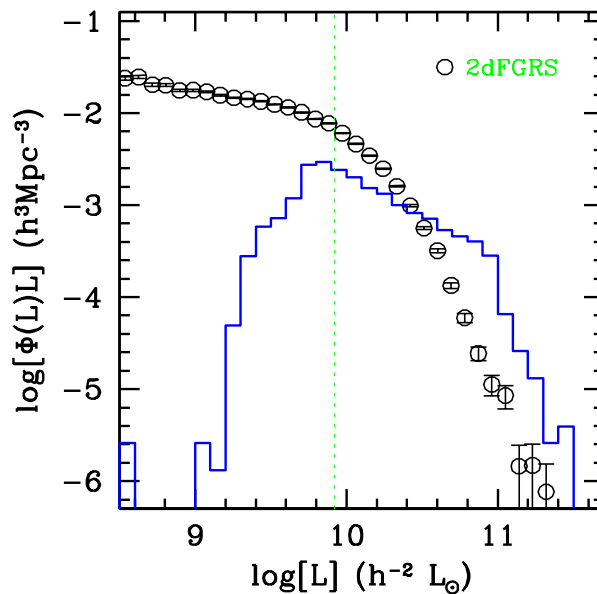
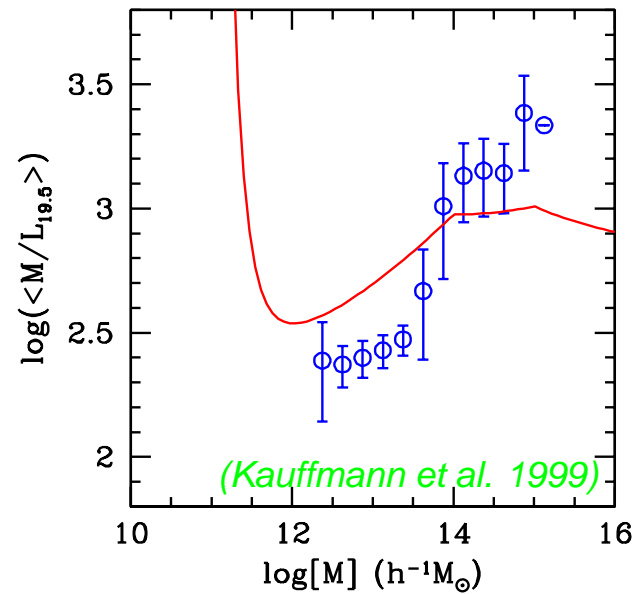
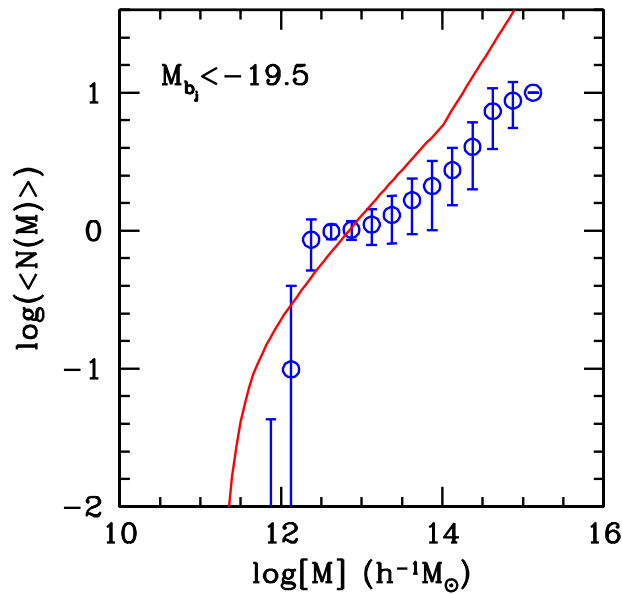
$$\Omega_m = 0.3; \Omega_\Lambda = 0.7, h = 0.7, \sigma_8 = 0.9, n = 1.0$$



vdB, Yang & Mo, 2003, MNRAS, 340, 771

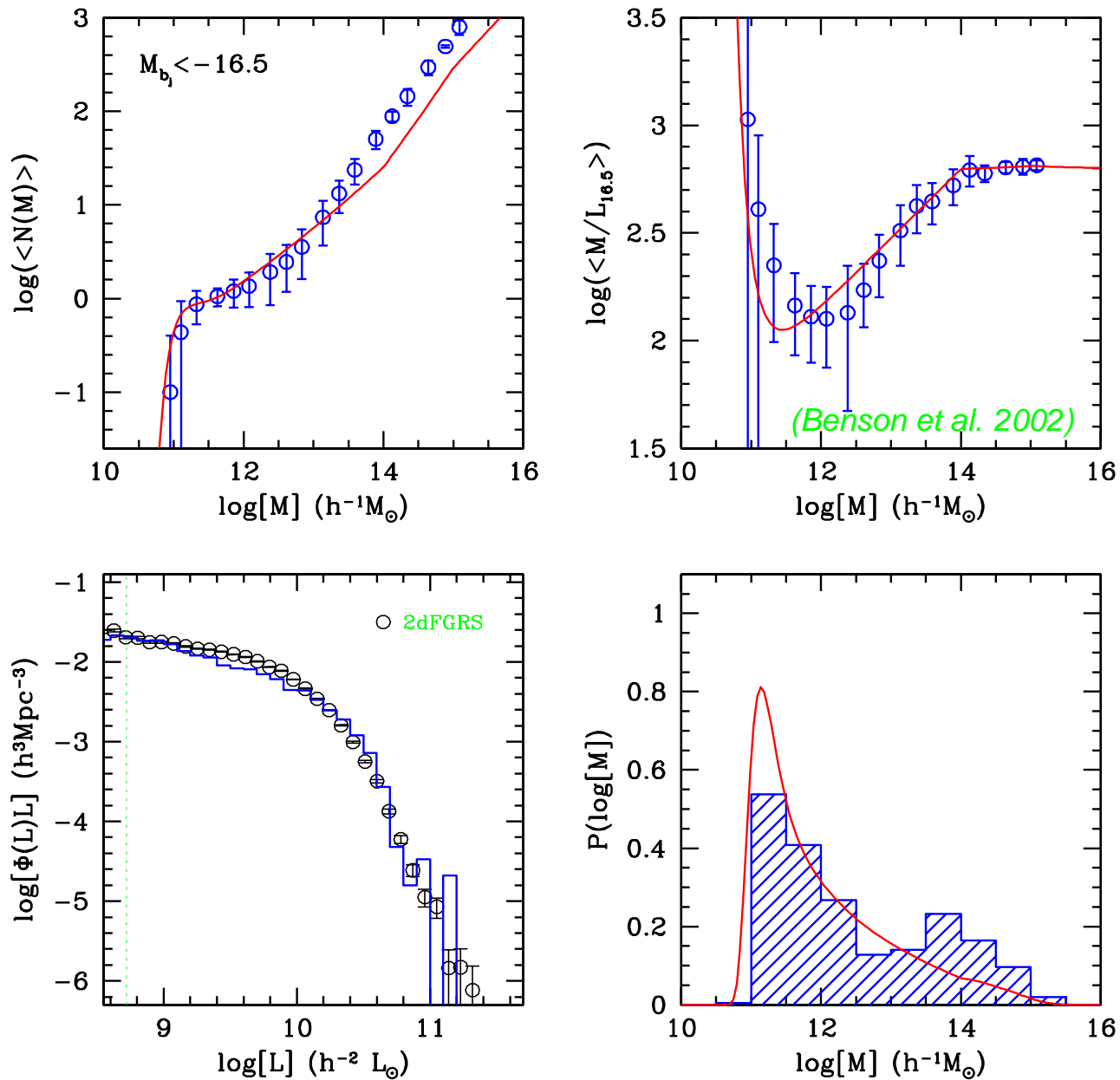
Concordance model fits $\Phi(L)$ and $r_0(L)$ of both early- and late-type galaxies.

Semi-Analytical Models I



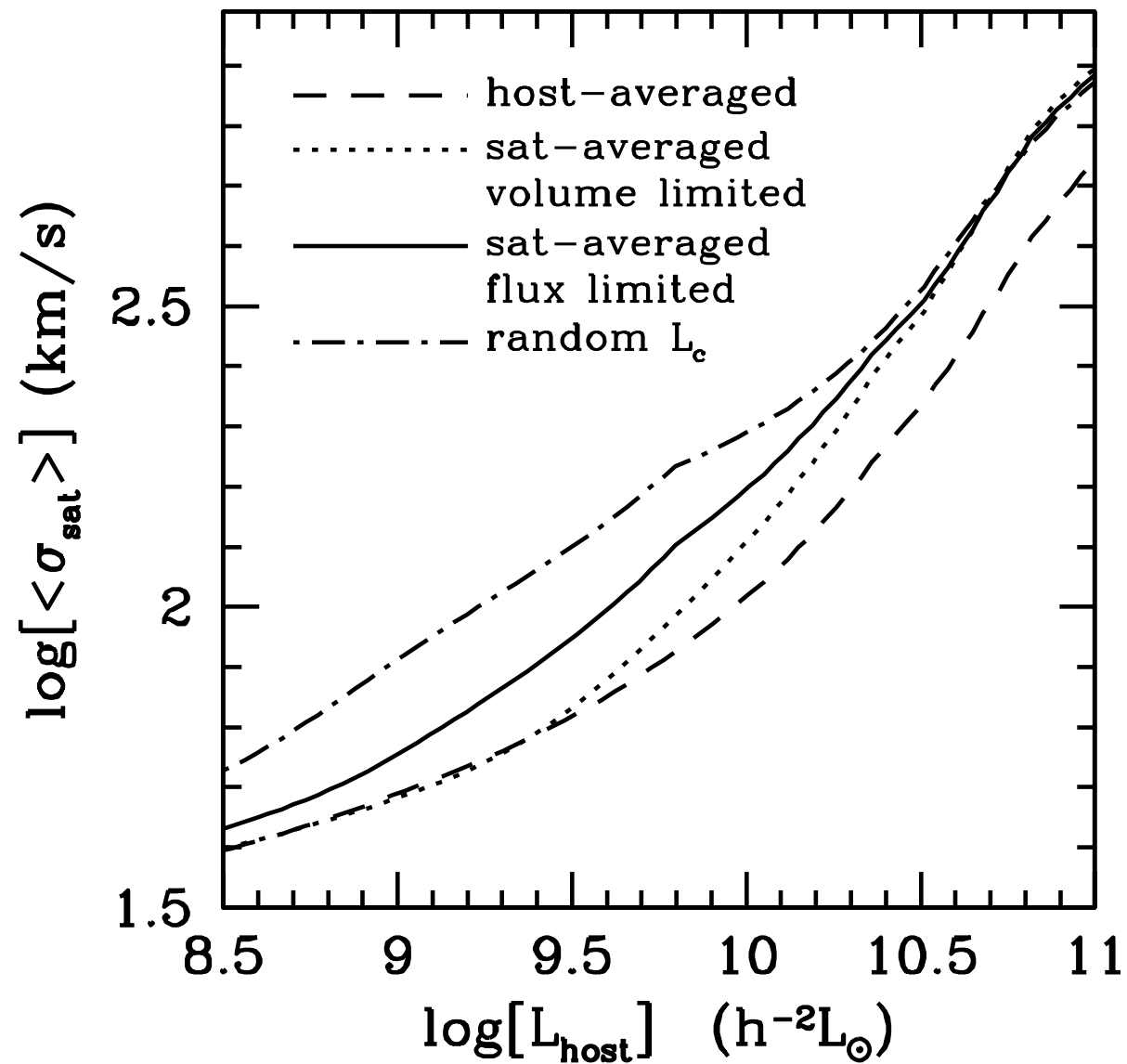
Poor agreement with CLF; but SAM doesn't fit LF

Semi-Analytical Models II



Good agreement with SAMs that fit LF

Bias in Satellite Kinematics



[back](#)

The Galaxy Correlation Function

The two-point galaxy-galaxy correlation function can be split in a 1-halo and a 2-halo term

$$\xi_{\text{gg}}(r) = \xi_{\text{gg}}^{1\text{h}}(r) + \xi_{\text{gg}}^{2\text{h}}(r) = \xi_{\text{gg}}^{1\text{h}}(r) + \bar{b}^2 \xi_{\text{hh}}^{2\text{h}}(r)$$

Here \bar{b} and $\xi_{\text{hh}}^{2\text{h}}(r)$ are computed as follows:

- $\bar{b} = \frac{1}{\bar{n}_g} \int_0^\infty n(M) \langle N(M) \rangle b(M) dM$ (Berlind & Weinberg 2002)
- $\langle N(M) \rangle = \int_{L_1}^{L_2} \Phi(L|M) dL$
- $\xi_{\text{hh}}^{2\text{h}}(r; M_1, M_2) = b(M_1) b(M_2) \xi_{\text{dm}}^{2\text{h}}(r)$ (Mo & White 1996)
- $\xi_{\text{dm}}^{2\text{h}}(r) = \xi_{\text{dm}}(r) - \xi_{\text{dm}}^{1\text{h}}(r)$ (Ma & Fry 2000)
- $\xi_{\text{dm}}(r) = \int_0^\infty \Delta_{\text{nl}}^2(k) \frac{\sin(kr)}{kr} \frac{dk}{k}$

NOTE: $\xi_{\text{gg}}^{1\text{h}}(r)$ can be ignored at large r

Derivation of Halo Bias II

According to **PS formalism** haloes are associated with regions with an overdensity $\delta > \delta_{\text{sc}}$

Therefore, we can compute $n(m, z|M, V) = n(m, z|\delta)$ by simply replacing δ_{sc} with $\delta_{\text{sc}} - \delta$ (**Peak-Background split**).

Using a Taylor Series expansion to first order, we write that

$$n(m, z|\delta) = n(m, z) + (\delta_{\text{sc}} - \delta - \delta_{\text{sc}}) \left(\frac{\partial n}{\partial \delta_{\text{sc}}} \right)_{\delta_{\text{sc}}}$$

which is sufficiently accurate as long as $\delta \ll \delta_{\text{sc}}$.

Substitution in eq. for δ_h and only keeping terms to lowest order in δ yields

$$\delta_h = \delta \left[1 - \frac{1}{n} \left(\frac{\partial n}{\partial \delta_{\text{sc}}} \right)_{\delta_{\text{sc}}} \right]$$

Derivation of Halo Bias III

According to the **PS formalism** the halo mass function is

$$n(m, z) = \sqrt{\frac{2}{\pi}} \frac{\bar{\rho}}{m^2} \left| \frac{d \ln \sigma}{d \ln m} \right| \sqrt{\nu} e^{-\nu/2}$$

with $\nu = \nu(m, z) = \delta_{\text{sc}}^2(z) / \sigma^2(m)$

If we define the **halo bias** as $b(m, z) = \delta_h / \delta$, one obtains that

$$b(m, z) = 1 + \frac{\nu - 1}{\delta_{\text{sc}}}$$

The characteristic mass m^* is defined by $\sigma(m^*) = \delta_{\text{sc}}$.

This implies that $\nu(m^*) = 1$, and thus that $b(m^*) = 1$

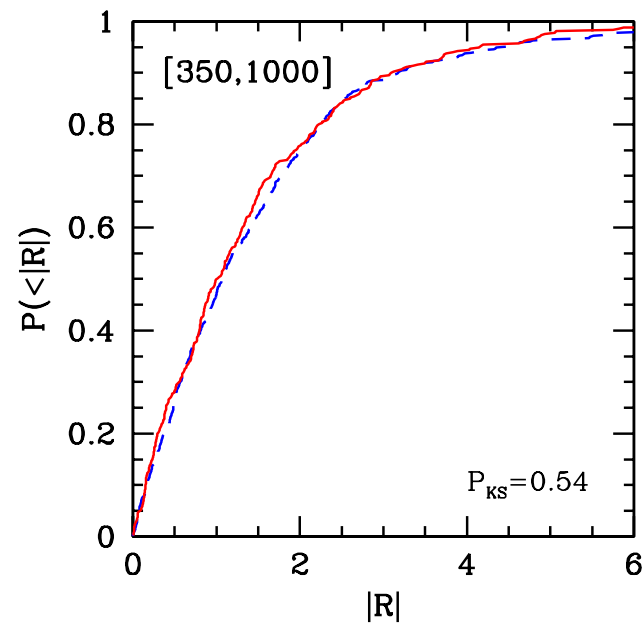
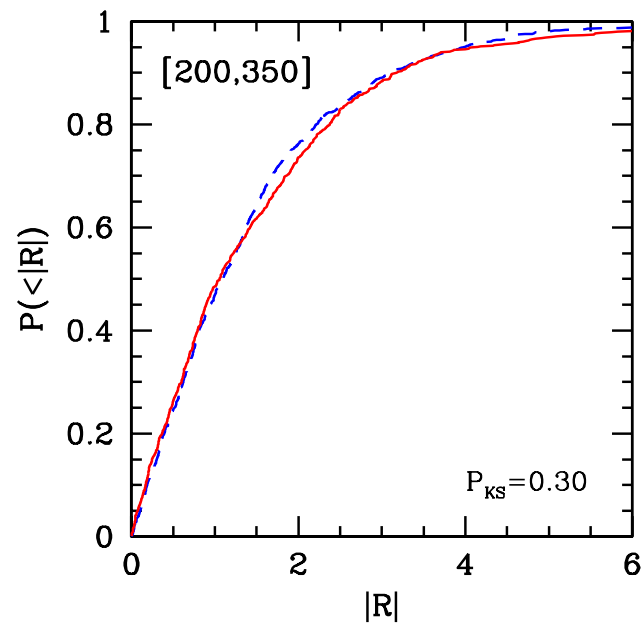
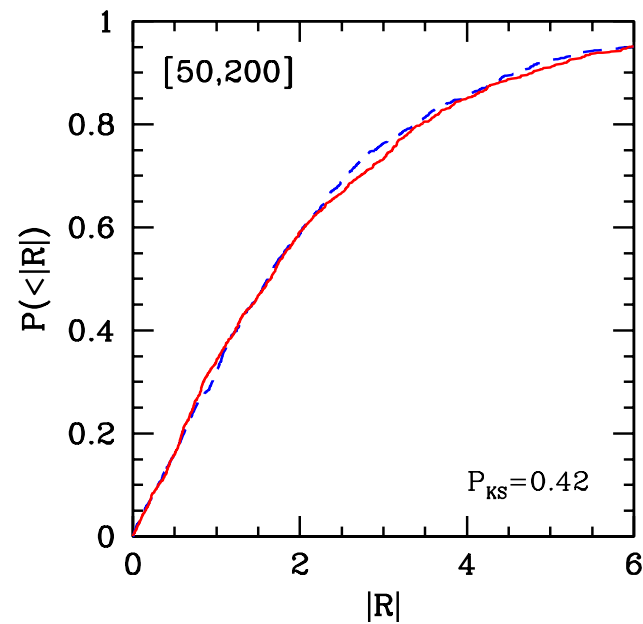
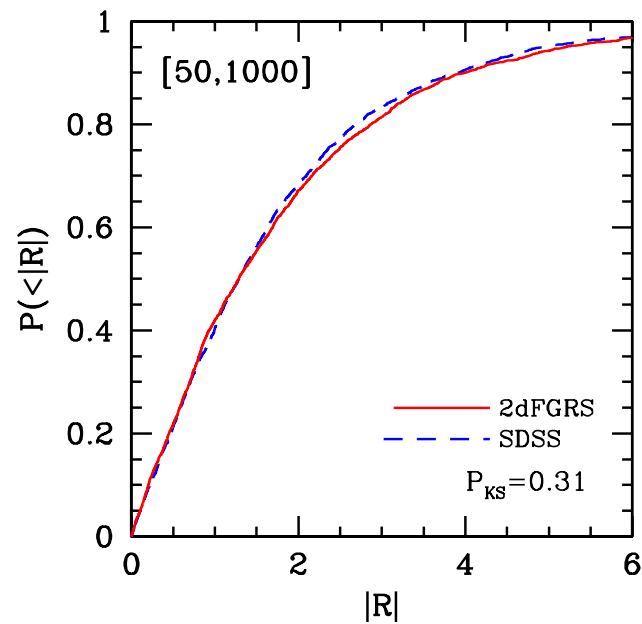
$$b(m, z) > 1 \quad \text{if } m > m^* \quad \text{(biased)}$$

$$b(m, z) = 1 \quad \text{if } m = m^* \quad \text{(unbiased)}$$

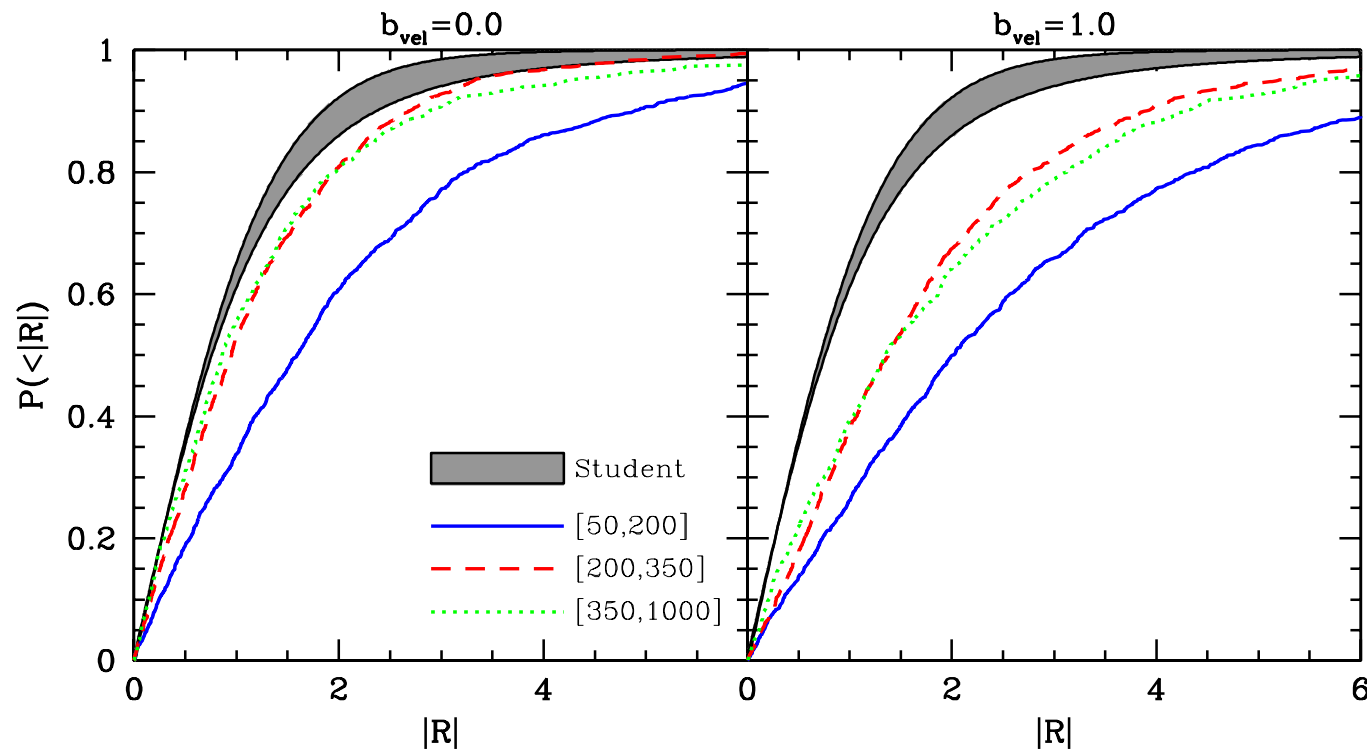
$$1 - \frac{1}{\delta_{\text{sc}}} < b(m, z) < 1 \quad \text{if } m < m^* \quad \text{(anti-biased)}$$

Note that there is an absolute minimum to the halo bias.

Comparing 2dFGRS with SDSS

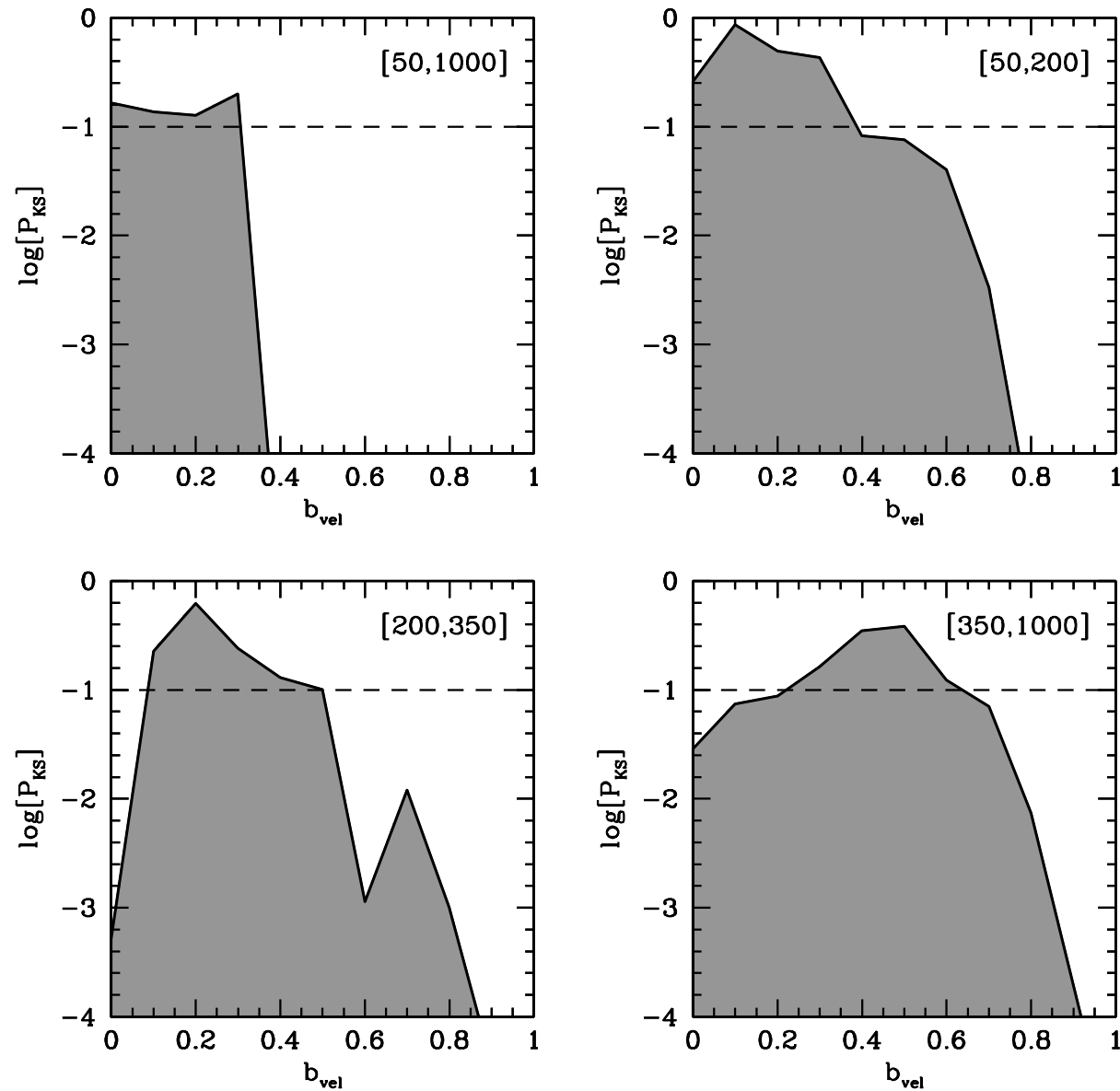


Testing \mathcal{R} -statistic with MGRSs



- We construct **ten MGRSs**, that only differ in the **velocity bias** (b_{vel}) of the brightest halo galaxy
- Due to **interlopers** and **incompleteness effects** $P(\mathcal{R})$ is significantly broader than **Student t-distribution**, even for $b_{\text{vel}} = 0$
- Different b_{vel} result in significantly different $P(\mathcal{R})$, allowing for a **determination** of b_{vel} despite interlopers and incompleteness.

Mass Dependence of b_{vel}



The velocity bias b_{vel} is larger for more massive haloes, in agreement with a **hierarchical formation scenario**.

Assumptions

- All dark matter is collapsed into dark matter haloes
- All galaxies live in dark matter haloes
- Central galaxy resides at rest at center of halo
- Satellite galaxies follow dark matter distribution (NFW profile)
- $P(N_{\text{sat}}|M)$ is Poissonian
- Halo bias: $b = b(M)$; HOD: $P(N) = P(N|M)$
- Accuracy of $b(M)$, $n(M)$, $\rho(r)$ and $\xi_{\text{dm}}(r)$
- Halo is a cow, and therefore spherical
- What can we learn from HOD as function of redshift?

Assembly Bias: does it matter?

Assume that **halo bias** $b = b(M, c)$ with c the **halo concentration**, but that **HOD** does not depend on halo concentration, i.e., $\langle N \rangle = \langle N \rangle(M)$

$$b_g = \frac{1}{n_g} \int dM \langle N \rangle(M) \int dc b(M, c) n(M, c)$$

However, when ignoring this **assembly bias** we compute

$$b'_g = \frac{1}{n_g} \int dM \langle N \rangle(M) b(M) n(M)$$

Since $b(M) = \frac{1}{n(M)} \int b(M, c) n(M, c) dc$ one has that

$$b'_g = \frac{1}{n_g} \int dM \langle N \rangle(M) \int dc b(M, c) n(M, c) = b_g$$

However, when $\langle N \rangle = \langle N \rangle(M, c)$ this is no longer true, and $b'_g \neq b_g$

Important questions:

- What is $b(M, c)$?
- Do equal-mass haloes with different c have different galaxy properties?

Assembly Bias??

